LETTER TO THE EDITOR

REPLY TO "LETTER TO THE EDITOR," *FUSION TECHNOL.*, 27, 348 (1995).

This is our reply to the letter to the editor given in Ref. 1, which was published in Fusion Technol., **27**, *348 (1995). This letter discussed deep Dirac levels. The following text was sent on August 27, 1994 to the editor of* Fusion Technology *with our request to publish it together with Ref. 1.*

We agree with the Ref. 1 authors, Rice, Kim, and Rabinowitz (R-K-R), that this is an important scientific matter, and we sincerely desire to resolve this matter in a collegial discussion. We would have preferred to do this in a direct, personal exchange of letters with R-K-R prior to publication. However, R-K-R used this forum, and we have no other choice than to respond in like manner.

R-K-R use Eq. (1) in Ref. 1 for the radial relativistic Schroedinger equation in the form

$$\frac{d^2\chi}{d\rho^2} + \left[\frac{\gamma^2}{\rho^2}\right] = 0 \ . \tag{1}$$

The last term of Eq. (1) should be multiplied by the wave function χ, which is probably a typographical error. However, Eq. (1) is not a relativistic Schroedinger equation. The correct relativistic Schroedinger equation [derived from Eq. (51.15) of Schiff[2] if we introduce $R(\rho) = \chi\alpha/\rho$ and use all other notation from Ref. 2] is

$$\frac{d\chi^2}{d\rho^2} + \left[\frac{\lambda}{\rho} - \frac{1}{4} - \frac{l(l+1) - \gamma^2}{\rho^2}\right]\chi = 0 \ . \tag{2}$$

Note that the correct Eq. (2) contains the parameter λ, from which we can calculate the energy levels E and the correct wave functions. On the other hand, Eq. (1), as used by R-K-R in Ref. 1, has three missing terms, resulting in the impossibility of calculating energy (missing the λ term) and calculating the wave function (the wave function must be calculated using an equation valid from $0 < r < \infty$ to normalize it). Therefore, the derived Eqs. (2) through (6) in Ref. 1 are irrelevant because they do not solve the case of bound electrons. In addition, Eqs. (1), (4), and (5) in Ref. 1 are approximations only, and therefore, Eq. (6) in Ref. 1 is also approximately valid ($1 + s_+ \sim 1 + 0.00005 \sim 1$).

Equation (2) is the Whittaker equation (13.1.31) in Ref. 3:

$$\frac{d^2w}{dz^2} + \left[-\frac{1}{4} + \frac{\kappa}{z} + \frac{\left(\frac{1}{4} - \mu^2\right)}{z^2}\right]w = 0 \ , \tag{3}$$

where

$$\begin{aligned} z &= \rho \ , \\ w &= \chi \ , \\ \kappa &= \lambda \ , \\ \tfrac{1}{4} - \mu^2 &= \gamma^2 - l(l+1) \\ \mu &= +[\tfrac{1}{4} - \gamma^2 + l(l+1)]^{1/2} \ , \end{aligned}$$

or

$$\mu = -[\tfrac{1}{4} - \gamma^2 + l(l+1)]^{1/2} \ .$$

The solution of Eq. (3) is given in Eq. (13.1.32) of Ref. 3:

$$\begin{aligned} M_{\kappa,\mu}(z) &= e^{-z/2} z^{1/2+\mu} {}_1F_1(a,b;z) \\ &= e^{-\rho/2}\rho^{s+1} {}_1F_1[s+1-\lambda, 2(s+1); \rho] \ , \end{aligned} \tag{4}$$

where

$$\begin{aligned} a &= \tfrac{1}{2} + \mu - \kappa = s + 1 - \lambda \ , \\ b &= 1 + 2\mu = 2(s+1) \ , \\ s &= s_+ = -\tfrac{1}{2} + [\tfrac{1}{4} - \gamma^2 + l(l+1)]^{1/2} \ , \end{aligned}$$

and

$$s = s_- = -\tfrac{1}{2} - [\tfrac{1}{4} - \gamma^2 + l(l+1)]^{1/2} \ .$$

The ${}_1F_1$ function is the Kummer power series having an infinite number of terms defined in Eq. (13.1.2) of Ref. 3. Before we introduce the boundary conditions at $\rho = \infty$, we can combine the general solution of Eq. (4) for both s_+ and s_- in the form

$$\chi(\rho) = C_1 e^{-\rho/2}\rho^{s_+ + 1} {}_1F_1[s_+ + 1 - \lambda, 2(s_+ + 1); \rho] + C_2 e^{-\rho/2}\rho^{s_- + 1} {}_1F_1[s_- + 1 - \lambda, 2(s_- + 1); \rho] . \quad (5)$$

Both Kummer series in Eq. (5) reach infinite value at $\rho \to \infty$ because both series are approaching e^{ρ}. To avoid this problem, we have to terminate the Kummer series in Eq. (5), applying the termination conditions

$$a = s_+ + 1 - \lambda = -n' \quad \text{with } n' = 0,1,2,3,\ldots . \quad (6)$$

This condition will terminate only the first series in Eq. (5). We must set $C_2 = 0$ in Eq. (5) to allow the normalization [the second series remains infinite even with condition (6)]. We have another equally valid termination condition for the second Kummer series in Eq. (5):

$$a = s_- + 1 - \lambda = -n' \quad \text{with } n' = 0,1,2,3,\ldots . \quad (7)$$

This termination condition will terminate the second series but not the first series, and we must set $C_1 = 0$ in Eq. (5) to allow the normalization [the first series remains infinite even with condition (7)].

The value n' defines the number of terms in the terminated Kummer series. For a case of $l = 0$, $n' = 0$, and the terminated Kummer series, is simply equal to 1. From Eqs. (5), (6), and (7), we can see that for $l = 0$, the solution is either

$$\chi(\rho) = C_1 e^{-\rho/2}\rho^{s_+ + 1}$$
$$[\text{with termination condition (6) and } C_2 = 0] \quad (8)$$

or

$$\chi(\rho) = C_2 e^{-\rho/2}\rho^{s_- + 1}$$
$$[\text{with termination condition (7) and } C_1 = 0] . \quad (9)$$

These solutions cannot be linearly combined as suggested in Ref. 1. Note that our solution (9) is only superficially similar to the solutions in Eq. (4) of Ref. 1, which were derived from the incorrect Eq. (1) and which did not include any termination conditions. This is why we insist now and in Ref. 4 that two solutions, each calculated with different termination conditions (6) or (7) and having a different eigenvalue E, cannot be combined.

In conclusion, we believe that the mathematical arguments of Refs. 1, 5, and 6 about the nonexistence of deep Dirac levels are completely in error. We agree with Ref. 1 that there should exist the fusion reaction products at some rate resulting from the nuclear reactions of deep Dirac level atoms with other nuclei. We discuss this matter in Ref. 7.

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7. J. A. MALY and J. VÁVRA, "Electron Transitions on Deep Dirac Levels II," *Fusion Technol.*, **27**, 59 (1995).

Note from the Editor: Because of an error, this letter was not published when the original letter to which it refers appeared [*Fusion Technol.*, **27**, 348 (1995)]. We apologize for any inconvenience caused by this inadvertent oversight.