For the fourth-order moments, we write

$$\frac{dG_{22}}{dt} + (1 + a - c_2)\, G_{22}(t) = \frac{4}{5}\, G_{11}(t) \tag{8a}$$

$$\frac{dG_{31}}{dt} + (1 + a - \bar{\mu})\, G_{31}(t) = G_{20}(t) + 2\, G_{22}(t) \tag{8b}$$

$$\frac{dG_{40}}{dt} + aG_{40}(t) = 4\, G_{31}(t) \quad . \tag{8c}$$

The integrations are straightforward, although increasingly tedious:

$$G_{22}(t) = \frac{4\exp(-at)}{15(1-\bar{\mu})}\left\{\frac{1-\exp[-(1-c_2)t]}{1-c_2} - \frac{\exp[-(1-\bar{\mu})t] - \exp[-(1-c_2)t]}{\bar{\mu}-c_2}\right\} \quad , \tag{9a}$$

$$G_{31}(t) = \frac{2\exp(-at)}{3}\left(\frac{1}{(1-\bar{\mu})^3}\{[2+(1-\bar{\mu})t]\exp[-(1-\bar{\mu})t] - 2 + (1-\bar{\mu})t\} + \frac{4}{5}\,\frac{\{1-\exp[-(1-\bar{\mu})t]\}}{(1-c_2)(1-\bar{\mu})^2} - \frac{4t\exp[-(1-\bar{\mu})t]}{5(1-\bar{\mu})(\bar{\mu}-c_2)} \right.$$
$$\left. + \frac{4}{5(1-c_2)}\left\{\frac{\exp[-(1-\mu)t] - \exp[-(1-c_2)t]}{(\mu-c_2)^2}\right\}\right) \quad , \tag{9b}$$

and

$$G_{40}(t) = \frac{8\exp(-at)}{3}\left[\frac{3 - 2\xi + \frac{1}{2}\xi^2 - (3+\xi)\exp(-\xi)}{(1-\bar{\mu})^4} + \frac{4}{5(1-\bar{\mu})^3}\left[\frac{\exp(-\xi) - 1 + \xi}{1-c_2} - \frac{1-(1+\xi)\exp(-\xi)}{\bar{\mu}-c_2}\right.\right.$$
$$\left.\left. + \frac{4\Big((1-c_2)[1-\exp(-\xi)] - (1-\bar{\mu})\{1-\exp[-(1-c_2)t]\}\Big)}{5(1-c_2)^2(\bar{\mu}-c_2)^2(1-\bar{\mu})}\right]\right] \quad , \tag{9c}$$

where $\xi \equiv (1-\bar{\mu})\,t$.

The three-dimensional moments may be found most easily from the plane-point transformation.[2] The distribution from a point source $\Phi(r)$ in terms of the distribution from a plane source $\phi(x)$ is

$$\phi(r) = -\frac{1}{2\pi r}\,\phi'(r) \quad , \tag{10}$$

and the radial moments,

$$M_{2n}(t) = \int \phi(r)\, r^{2n} \cdot 4\pi r^2\, dr \quad ,$$

are simply

$$M_{2n}(t) = (2n+1)\, G_{2n,0}(t) \quad .$$

Then, for short times, $t \ll 1$, we have $M_4(t) \to t^4$, and for $t \gg 1$, the distribution of the unabsorbed neutrons is

$$\langle R^4(t)\rangle \to \frac{20}{3(1-\bar{\mu})^2} \times \left[t^2 - \frac{4t}{1-\bar{\mu}} + \frac{6}{(1-\bar{\mu})^2} + \frac{8}{5(1-c_2)}\left(t - 1 - \frac{1-\bar{\mu}}{1-c_2}\right)\right] . \tag{11}$$

The mean fourth moment to absorption can be calculated from Eq. (9c) and is given by

$$\langle R^4\rangle = \int_0^\infty 5\, G_{40}(t)\, a\, dt = \frac{8[5(1-c_2) + 9a]}{3a^2(1-\mu+a)^2(1-d_2+a)} \quad , \tag{12}$$

which agrees with Eq. (4c).

The higher moments can be similarly calculated, but there is probably no need for exhibiting the results explicitly; the recursive formulas are easily converted to numerical calculation.

E. Richard Cohen

Science Center, Rockwell International
P. O. Box 1085
Thousand Oaks, California 91360

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[2]B. DAVISON and J. B. SYKES, *Neutron Transport Theory*, p. 64, Oxford University Press (1958).

Reply to "Comments on 'Exact Time-Dependent Second Spatial Moment of the One-Speed Neutron Transport Model' "

Cohen's approach to calculating the time-dependent moments by starting with the transport equation is elegant and efficient. I suppose that the simplicity of the second moment formula should lead one to suspect that it must be the solution to some simple differential equation. This, in turn, should suggest that closed systems of moments equations might be obtained—a rare and rewarding occurrence.

Cohen mentions that "it is unnecessary to calculate the full distribution" to find spatial moments. I assume he means, speaking probabilistically, that it is unnecessary to find the probability density function that describes the neutron's distance from the origin at time t; or, in transport theory terms, that it is unnecessary to find an expression for the neutron number density as a function of radius and time. He is surely correct, and I certainly did not find, or attempt to find, the probability density function in question. To do so would mean that the entire point-source, time-dependent problem would be solved. In effect, Case and Zweifel[1] have solved the complete problem for the infinite one-dimensional case. They find the Green's function for the monodirectional plane source of neutrons at time zero problem. Their solution is somewhat formal and involves integrals of complex valued function.

For those who are facile with the transport equation, several comments and questions might be of interest:

1. Is it possible to extract the time-dependent spatial moments from the Case and Zweifel solution?
2. Can the transport equation approach shed any light on the odd time-dependent spatial moments? The odd moments are zero in the one-dimensional case but

[1]K. M. CASE and P. F. ZWEIFEL, *Linear Transport Theory*, pp. 186-187, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts (1967).

surely are not zero in the three-dimensional case since the neutron distance from the origin can only take on positive values. In my hands, the random walk approach quickly gets into trouble in even attempting to find the first moment.

3. Considerable work was required to find the second spatial moment via the random walk approach, but little work was required to find information about the neutron position at the n'th collision. Can the transport equation approach yield information about the expectation of the square of the neutron's distance from the origin at the n'th collision?

C. S. Barnett

Lawrence Livermore Laboratory
P. O. Box 808
Livermore, California 94550

July 23, 1973

Corrigendum

J. C. TURNAGE, "A New Method of Solving the Multimode Reactor Kinetics Equations," *Nucl. Sci. Eng.*, 51, 67 (1973).

In Table I, the entries for $\nu\Sigma_f$ and for Σ_a in regions II, III, and IV should be 0.0194962, rather than 0.194962 as reported. Thus, for the one-group problems, it should have been reported that in case I, Σ_a was changed from 0.0194962 to 0.0210 cm^{-1}, that in case II, Σ_a was varied from 0.0194962 to 0.0185001 cm^{-1} in 1 sec, and that in case III, Σ_a was changed from 0.0194962 to 0.0190472.

In addition, the parameter defined in Table VIII as $\nu\Sigma_a^2$ should have been given as $\nu\Sigma_f^2$.