Letter to the Editor

Editorial Comment: We are pleased to present the following Letter from Professor Nikolaev bringing the attention of our readers to the development in the USSR of the subgroup method for cross-section evaluation. In supplemental correspondence, Professor Nikolaev has pointed out that the subgroup method is identical to the method of probability tables developed independently in the U.S., without knowledge of the earlier work in the USSR due to an unfortunate lack of communication. D. C.

Comments on the Probability Table Method

The so-called probability table method for treating resonance self-shielding in the unresolved resonance region was proposed in 1972 by Levitt[1] for use in Monte Carlo calculations. In the recent work by Cullen,[2] the probability table method was proposed for solving the Boltzmann equation by deterministic methods.

With this object in mind, matrix boundary conditions characteristic of the method discussed were given in Ref. 2. The probability table method is nothing but the "subgroup method," the idea of which was briefly stated in 1963 (Ref. 3). In further development of the subgroup method, a series of results was obtained, some of which are very important for practical applications, although not mentioned in Refs. 1 and 2.

The problem of boundary conditions is complicated from the point of view of a practical solution of the Boltzmann equation by the subgroup method. Cullen's boundary conditions can be used for calculation of the multiregion systems only if

1. the homogeneous regions of the calculated system have a rather large optical thickness
2. the adjacent regions do not contain the same nuclides, and thus if resonance structures of neutron fluxes in those regions are not correlated.

More general boundary conditions for neighboring media were formulated in Refs. 4 and 5. Proposed in Ref. 6 were the practical methods of taking into account the correlations in the cross-section structure in different regions when neutrons have a noticeable probability of passing through one or several regions.

An optimal description of total cross-section probability density by the sum of weighed δ functions allows a sharp reduction in the number of subgroups as compared to the Levitt method of calculation of integrals over the total cross section by dividing the integrated region into equal intervals $\Delta\Sigma_{tot}$, as well as in comparison with the method of dividing into equi-probable intervals proposed by Cullen.

In Ref. 7, subgroup parameters were obtained from such momenta as

$$\langle(\sigma_{tot} + \sigma_0)^n\rangle \quad \text{and} \quad \langle\sigma_r(\sigma_{tot} + \sigma_0)^n\rangle \quad .$$

These momenta for $n = 0, -1, -2$ were restored from the Bondarenko self-shielding factors listed in Ref. 8. The temperature dependence of subgroup probabilities and of cross sections was described parametrically.

In Ref. 9, an analytical method of determining the subgroup parameters (multiband probabilities and cross sections) through the momenta $\langle\sigma_{tot}^n\rangle$ and $\langle\sigma_r\sigma_{tot}^n\rangle$ at $n = -4, 3, \ldots, 3$ was developed. This method proved to be more effective.

The experience obtained in calculations with the help of these methods shows that in all cases of describing the neutron cross-section structure by subgroups, it is possible to restrict ourselves to no more than four subgroups. For example, when the model probability distribution function cited by Cullen[2] is described by two subgroups ($\rho_1 = 0.6363$, $\sigma_{tot,1} = 11.634$, and $\rho_2 = 0.3637$, $\sigma_{tot,2} = 7.142$), the accuracy of the calculated transmission function appears considerably higher than when ten subgroups determined by Cullen's method are used.

Preparation of data for subgroup calculations can be carried out more effectively than by the methods described in Refs. 1 and 2. For example, for the calculation of momenta, we use[10] quasi-random consequences of 4 or 20 resonances,[11] which allows us to make averages of width and distance distributions more effectively than is possible with the help of stochastic ladders.

For practical calculations, we now use the 26-group

[1]L. B. LEVITT, *Nucl. Sci. Eng.*, **49**, 450 (1972).

[2]D. E. CULLEN, *Nucl. Sci. Eng.*, **55**, 387 (1974).

[3]M. N. NIKOLAEV and V. V. PHILIPPOV, *At. Energ.*, **15**, 493 (1963).

[4]M. N. NIKOLAEV and A. A. IGNATOV, *Bull. Informazionnogo Zentra po Jadernym Dannym*, **3**, 409, Atomisdat, Moscow (1966).

[5]M. N. NIKOLAEV et al., *At. Energ.*, **29**, 11 (1970); see also, *At. Energ.*, **30**, 426 (1971).

[6]M. N. NIKOLAEV and D. A. USIKOV, *At. Energ.*, **34**, 112 (1973).

[7]M. N. NIKOLAEV and V. F. KHOKHLOV, *Bull. Informazionnogo Zentra po Jadernym Dannym*, **4**, 420, Atomisdat, Moscow (1967); see also, Report on the English-Soviet Seminar Nuclear Data for Reactor Calculations, Dubna, June 18-22, 1968.

[8]L. P. ABAGJAN, N. O. BAZAZJANTZ, I. I. BONDARENKO, and M. N. NIKOLAEV, *Group Constants for Reactor Calculations*, New York Consultants Bureau (1964).

[9]V. V. SINITZA and M. N. NIKOLAEV, *At. Energ.*, **35**, 429 (1973).

[10]L. P. ABAGJAN and M. N. NIKOLAEV, *Jaderno-Physicheskie Issledovanija in SSSR*, **15**, 27, Zentra po Jadernym Dannym in Obninsk, Atomisdat, Moscow (1973).

[11]M. N. NIKOLAEV et al., *Proc. Soviet-Belgium-Netherlands Sem. Some Problems of Reactor Physics*, Melekess, Feb. 1970, Rep. D-19, ZNII-ATOMINFORM, Moscow (1970); see also, INDC(CCP)-16/L, p. 27, International Atomic Energy Agency, Vienna (1971).

cross-section set with the subgroup representation of a resonance structure.[7,12] The data of this set are retrieved by the ARAMACO (Ref. 12) or ARMONT (Ref. 13) computer codes. In those codes, it is possible to convolute (for special calculation) into one "generalized nuclide" all the nuclides that exist only in one homogeneous region of a multiregion system or in different regions but in the same relative concentrations.

Convolution of subgroups is carried out under the condition of preservation of cross-section momenta by the method described in Ref. 9. The catalog of subgroup constants set ARAMACO for 44 nuclides has been sent to the International Atomic Energy Agency for international exchange.

Experimental determination of subgroup parameters in the unresolved resonance region does not demand a high resolution; they may be obtained by the analysis of neutron transmission curves measured up to large attenuation of a primary beam by the flat response detectors and by the self-detection method.[3] In this way, rich information on the total cross-section structure of many nuclides has been already obtained.[14-16]

Unfortunately, measurements of transmission curves by the self-detection method were carried out only for relatively small attenuations,[17,18] and it was very difficult to obtain from the data partial subgroup cross sections.

The subgroup data, evaluated by the analysis of transmission curves or by calculations, can be included into the evaluated data library SOCRATOR (Ref. 19), in which special formats are stipulated for storing such information.

Application of subgroup parameters for calculation of space-independent self-shielded cross sections of homogeneous media appears more effective than the use of the Bondarenko factors. In the group constants set[8] assigned for calculations in the P_1-approximation, self-shielding factors are calculated by averaging over the spectra $1/(\Sigma_{tot} + \Sigma_0)$ for the zeroth mode and $1/(\Sigma_{tot} + \Sigma_0)^2$ for the first mode. [Cullen's statement that averaging on spectrum $1/(\Sigma_{tot} + \Sigma_0)$ only, used in Ref. 8, is incorrect.] For calculations in higher approximations (P_N, for example), it is desirable to use averaging over the spectra considering the terms with $1/(\Sigma_{tot} + \Sigma_0)^n$ and such momenta as $\langle \Sigma_{sl}/\Sigma_{tot} + \Sigma_0)^n \rangle$ for $0 \leq 1 \leq N$, $1 \leq n \leq N+1$ (Ref. 20).

Application of subgroup parameters allows us to calculate the results of averaging over these spectra without increasing the input data volume, as in the case of description of cross-section structures by self-shielding factors. Such an algorithm is realized in the group constant processing codes ARAMACO (Ref. 12) and ARMONT (Ref. 13).

Experience of practical application of the subgroup method has shown that:

1. It involves a high calculational accuracy not only in the unresolved resonance region but in many cases when cross-section structure in the vicinity of wide scattering resonances [for example, resonances of ^{56}Fe (Refs. 5 and 21)] is described. In particular, this algorithm is very convenient for describing deep neutron penetration across the shielding through the interference minima in the total cross section.

2. When solid shielding is calculated, the use of the subgroup method for calculation of the first collision distribution and treating the consequent neutron transport in the terms of space-independent self-shielded group cross sections allows us to calculate transmission of neutrons across the shielding and albedo with a rather high accuracy.[5,22]

3. When multiregion systems are calculated the use of "through subgroups"[6] for describing the correlations of cross-section structures in different regions, difficulties can appear to be caused by the large number of subgroups that are necessary to be taken into consideration. These difficulties can be obviated by the use of subgroup constants for "generalized nuclides"[13] and/or by the algorithm of "degeneration" of subgroups described in Refs. 23 and 24.

4. The algorithm for solving the adjoint transport equation in the subgroup approximation by the Monte Carlo method is described in Ref. 25. Note that the use of the subgroup method is a way for solving the famous problem of what the weight function must be for averaging the group constants for conservation of not only k_{eff} but also of such functions as neutron lifetime and so on.[26]

5. In Refs. 27 and 28, the application of the subgroup method for heterogeneous lattice cells calculation by the collision probability method is described.

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