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LETTERS TO THE EDITORS

Heating of Positive Ions in a Thermonuclear Plasma

The question of how the positive ions of a thermonuclear plasma get their large observed energies in the short duration of the discharge is raising considerable perplexity at present. The mechanism of collision with electrons apparently is inadequate, as shown by Spitzer (*1*). Some mechanism involving oscillation has been suggested by Spitzer and others.

It seems reasonable to assume that the main energy input to the discharge occurs through the presence of a beam of high velocity electrons accelerated by the electromotive force of the external energy source. One possible mechanism for the transfer of energy from this primary beam and storage of energy in the underlying plasma is the excitation of waves in the plasma whose group velocity relative to the tube is zero or small.

It is well known that, for a one-velocity one-component plasma in which the positive ions are treated as a fixed space charge, the frequency of oscillations is independent of wave length in a frame of reference moving with the drift velocity of the electrons. The group velocity therefore is zero in this frame. Such oscillations thus will not provide the sought-for mechanism. It is true that, if the electrons are allowed to have a velocity spread, the group velocity is no longer zero (*2*), but the degree of dispersion provided by a reasonable assumption of the width of the electron velocity band does not give a group velocity large enough for the present purpose.

The above result, however, depends on the assumption that the radius of oscillating plasma is infinite, or at least very large compared with the wavelength of the oscillations. In the opposite case the result is quite different. It can be shown that, for a traveling wave with the space-time dependence $\exp i(\omega t - \gamma x)$ in a cylindrical plasma of radius a,

$$\omega - u\gamma = \omega_p\sqrt{1 - \gamma a K_1(\gamma a)}$$

where u is the drift velocity of the electrons, K_1 is the Bessel function of the second kind with pure imaginary argument, and

$$\omega_p = \sqrt{4\pi e^2 n/m}$$

is the plasma frequency, n being the number of electrons per unit volume. From this we get for the group velocity v_g

$$\begin{aligned} v_g = d\omega/d\gamma &= u[1 + a(\omega_p/u)G(a\gamma)] \\ &= u[1 + a\sqrt{2\pi e n/V}G(a\gamma)] \end{aligned}$$

where

$$G(a\gamma) = (1/2)a\gamma K_0(a\gamma)[1 - a\gamma K_1(a\gamma)]^{-1/2}, \quad v = mu^2/2e$$

so that v is the energy-equivalent of u in electron volts. If n is in units of 10^{14} cm^{-3}, v in kev, and the sign of γ is reversed so that the direction of propagation of the wave is opposite to that of the electron drift, this becomes

$$v_g = u[1 - 286\, a\sqrt{n/v}G(a\gamma)]$$

The function $G(a\gamma)$ is zero for large $a\gamma$, is equal to

$$[(1/2)\ln(1/a\gamma)]^{1/2}$$

for small $a\gamma$, and is 1/286 for $a\gamma = 6.04$.

Thus the group velocity is zero for $n = 10^{13}$ cm^{-3}, $v = 0.1$ kev, $a = 10$ cm, and a wavelength of 10.4 cm. These numbers seem appropriate for a thermonuclear plasma. On the other hand, for traveling wave tubes, in which a is a few millimeters, n is of the order 10^{11} cm^{-3}, and v is 10 kev, the difference $v_g - u$ would be negligible, which is why the difference can be neglected for such tubes.

It can also be shown from the dispersion equation for a two-component plasma, in which the motion of the positive ions is considered, that there exists another type of oscillation having strong dispersion, for which the group velocity can be zero or small. The phase velocity of this oscillation is close to the drift velocity of the positive ions, not of the electrons, and the ratio of amplitude of the oscillation of positive space charge to that of negative space charge can be large.

Collisions with neutral atoms, the principal mechanism for holding down positive ion velocities in ordinary plasmas, are absent in the fully ionized thermonuclear plasma. Therefore, there is no reason why the positive ions should not pick up a large drift velocity from the same emf that accelerates the electrons. We should therefore expect the final velocity distribution of the positive ions to be strongly anisotropic, with a large drift velocity on which are superposed random longitudinal velocities due to oscillations, and transverse random velocities of much narrower range. This explains the observations made in Zeta.

With this picture, the protons and deuterons will never reach a state of randomized velocities in the time of thermonuclear discharges of present duration, nor is it necessary that they should do so to achieve high neutron yields. The achievement of high yields and high energy conversion efficiencies depends on getting most of the energy of the primary source into the positive ions, and not on whether or not the velocity distribution of the positive ions is isotropic.

REFERENCES

1. L. SPITZER, *Nature*, **181**, 221 (1958).
2. BOHM AND GROSS, *Phys. Rev.* **75**, 1851 (1949).

H. M. MOTT-SMITH

Gibbs & Cox, Inc.
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