Letters to the Editor

Comments on "The Concept of Spatial Channel Theory Applied to Reactor Shielding Analysis"

The recent paper by Williams and Engle[1] is welcomed, since it gives a concise mathematical formulation and impressive sample demonstrations of widely known methods that have escaped such unifying documentation until now. The groundwork for the theory was laid by Case et al.[2] as early as 1953, with their equivalence theorem, allowing the reduction of any spatially finite problem to an infinite-medium problem. This theorem with its corollaries[2] is implicitly used by the authors in the above paper to formulate the theory. However, concluding their theory section (Sec. II), Williams and Engle make the following somewhat vague and misleading statements:

> . . . Eq. (8) can be thought of as the transport equation for the contributon flux:
>
> $$\nabla \cdot \boldsymbol{D}(\boldsymbol{r}) = S(\boldsymbol{r}) \ , \qquad (24)$$
>
> where
>
> $\boldsymbol{D}(\boldsymbol{r})$ = contributon current given by Eq. (15)
>
> $$S(\boldsymbol{r}) = \int_E \int_\Omega [\phi^+(\boldsymbol{\rho}), Q(\boldsymbol{\rho}) - \phi(\boldsymbol{\rho}), Q^+(\boldsymbol{\rho})] d\Omega dE$$
>
> = source of contributons. (25)
>
> Written in this form, the point reciprocity equation is merely the continuity equation for contributons. It differs from the Boltzmann transport equation by the absence of loss terms due to material interactions. This should be expected since contributons are never absorbed, and scattering events cannot be seen due to the integration over angle and energy.

These statements can lead the reader to believe that Eq. (24) is the basic equation from which the spatial distribution of the contributon current $\boldsymbol{D}(\boldsymbol{r})$ can be calculated. However, at close inspection, it is noticed that the source term $S(\boldsymbol{r})$, defined in Eq. (25), contains the phase-space distributions of the forward and adjoint fluxes, $\phi(\boldsymbol{\rho})$ and $\phi^+(\boldsymbol{\rho})$. To obtain these latter distributions, one must solve the forward and adjoint transport equations, and that solution, therefore, is a prerequisite to solving the above Eq. (24). However, once $\phi(\boldsymbol{\rho})$ and $\phi^+(\boldsymbol{\rho})$ are known, it seems much easier to calculate $\boldsymbol{D}(\boldsymbol{r})$ from its defining equation:

$$\boldsymbol{D}(\boldsymbol{r}) = \int_E \int_\Omega \boldsymbol{\Omega}\phi(\boldsymbol{\rho})\phi^+(\boldsymbol{\rho}) d\Omega dE \ , \qquad (15)$$

than by performing the integral of Eq. (25) and then solving Eq. (24). From these reasons, Eq. (24) *is not* a "transport equation for the contributon flux"; it is just the continuity equation for contributons in an implicit form.

In the final two sentences of their theory section, Williams and Engle emphasize that Eq. (24) does not contain any loss terms due to material interactions because contributons are never absorbed. Such argumentation can be misleading because the fact that contributons can never be lost in the process of streaming from source to detector does not mean that their distribution within the transporting medium is not influenced by the properties of this medium! In fact, we have, in a parallel development to the Williams and Engle paper, developed two equations for the contributon flux[3] that can be considered the real transport equations for contributons or "ψ-particles," as we named the product $\psi = \phi\phi^+$. For a purely absorbing medium, the monoenergetic transport equation for contributons in slab geometry has the form[3]

$$\mu^4\left(\frac{\partial^2\psi}{\partial x^2}\right)^2 + 4\mu^2 QR \frac{\partial^2\psi}{\partial x^2} - \mu^2\Sigma^2\left(\frac{\partial\psi}{\partial x}\right)^2 - 4\Sigma^2 QR\psi + 4Q^2R^2 = 0 \ , \qquad \text{(A)}$$

where $R \equiv Q^+$ in the Williams and Engle notation and Σ is the absorption cross section of the medium ($\mu = \cos\phi$, as conventionally used). Reference 3 gives also a second form of a transport equation for ψ when isotropic scattering is allowed.

The above equation, Eq. (A), is a "real transport equation for contributons" because it allows the calculation of ψ from first principles and does *not* require the solution of either the forward or adjoint form of the Boltzmann equation. The nonlinearity of Eq. (A) and other unusual features are further discussed in Ref. 3.

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[1] M. L. WILLIAMS and W. W. ENGLE, Jr., *Nucl. Sci. Eng.*, **62**, 92 (1977).

[2] K. M. CASE, F. deHOFFMANN, and G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, Vol. I, Chap. V, Los Alamos Scientific Laboratory (1953).

[3] S. A. W. GERSTL, "A New Concept for Deep-Penetration Transport Calculations and Two New Forms of the Neutron Transport Equation," LA-6628-MS, Los Alamos Scientific Laboratory (1976).

Response to "Comments on 'The Concept of Spatial Channel Theory Applied to Reactor Shielding Analysis' "

In reply to Gerstl's[1] comments concerning spatial channel theory,[2] the authors feel that his concern over referring to the contributon continuity equation [Eq. (24) in Ref. 2] as the "contributon transport equation" is more an argument over semantics than substance. Equation (24)

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[1] S. A. W. GERSTL, *Nucl. Sci. Eng.*, **64**, 798 (1977).

[2] M. L. WILLIAMS and W. W. ENGLE, Jr., *Nucl. Sci. Eng.*, **62**, 92 (1977).

expresses a contributon balance condition, just as the neutron transport equation expresses a neutron balance. In no way does it imply that contribution transport is not influenced by the material medium, since, as Gerstl points out, the contribution source is a function of the forward and adjoint fluxes.

The fact that the contributon conservation equation is not sufficient to solve for contributon density is not surprising. Considering a fluid flow analogy, it has been shown that Eq. (24) can be written in the same form as the mass continuity equation for compressible flow,[3] and that the equation is not sufficient to specify mass distribution. The energy and momentum equations must be considered as well.

Gerstl's statement that his Eq. (A) is the "true" contributon transport equation appeared to this reader to be rather bold. The fact that his mathematical manipulation (albeit very interesting) yields an equation for the product $\phi\phi^*$ does not necessarily mean that it is a transport equation, which implies certain physical characteristics. This reader could not determine the physical significance of a term such as

$$\left[\frac{\partial^2}{\partial x^2}(\phi\phi^*)\right]^2 ,$$

or how such a term pertains to transport phenomena.

Furthermore, Eq. (A) was derived only for the simplified case of monoenergetic neutrons in a purely absorbing medium, for which the forward and adjoint Boltzmann equations have analytic solutions. Therefore, the value for the contributon flux in this case can be computed on the back of an envelope, and there is little motivation for solving the second-order nonlinear equation developed by Gerstl.

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[3] M. L. WILLIAMS and W. W. ENGLE, Jr., "Spatial Channel Theory—A Technique for Determining the Directional Flow of Radiation Through Reactor Systems," *Fifth Int. Conf. Reactor Shielding*, Knoxville, Tennessee, April 18-22, 1977.

Optimized Taylor Parameters for Concrete Buildup Factor Data

Shure and Wallace[1] have provided very useful values of the parameters in Taylor's formula for gamma-ray buildup factors,

$$B_c = A \exp(-\alpha_1 \mu r) + (1 - A) \exp(-\alpha_2 \mu r) \quad , \qquad (1)$$

based on data recently generated by Eisenhauer and Simmons.[2] However, the values given in Ref. 1 do not provide the "best" fit in the sense of minimizing the maximum percent deviation quoted in the tables.[3]

To show that better values, in the Tchebycheff sense, can be obtained, we have calculated values for a representative set of photon energies from 0.04 to 15.0 MeV, on the basis of a point source of monoenergetic photons in an infinite medium of ordinary concrete and on the assumption of an exposure (called "dose" in Ref. 1) detector.[4] The same data of Eisenhauer and Simmons as were fitted for Ref. 1 were used here. The results are given in Table I.

It can be seen from the comparison in the table that the values of the parameters shown here appear to provide a somewhat better fit than those in Ref. 1. Presumably, the other tabulated data in that reference could likewise be improved.

One minor point needs explanation and comment. The fitting accomplished by us ignored the data at 0.5 mfp. In our judgment, this was a wise thing to do because such values are not reliable:

1. It is known that the moments method is less reliable at distances near a point source. See Table X of Ref. 2, for example.
2. At small values of penetration, the Eisenhauer-Simmons code is known to give, under certain circumstances, results that are so obviously spurious they have to be replaced by interpolated values.[5]

TABLE I

Taylor Parameters for Exposure Buildup Factor Data, Point Source in Infinite Concrete Medium (mfp range 0 to 40)

Source Energy (MeV)	Parameters for Eq. (1)			Maximum Percent Deviation (This Letter)	Corresponding Maximum Deviation (Ref. 1) (%)
	A	α_1	α_2		
0.04	2.33	-0.0147	0.317	4.5	6.14
0.06	5.29	-0.0414	0.210	5.3	7.32
0.08	18.3	-0.0382	0.0469	4.7	4.91
0.10	73.8	-0.0394	-0.0145	6.0	9.64
0.20	144	-0.0741	-0.0598	20.2	37.22
0.50	62.0	-0.0688	-0.0424	22.2	41.28
1.00	97.0	-0.0396	-0.0271	15.2	25.12
2.00	38.7	-0.0250	-0.00227	7.0	8.47
5.00	10.42	-0.0244	0.0269	1.5	2.14
10.00	5.10	-0.0269	0.0450	2.3	3.41
15.00	4.04	-0.0267	0.0393	2.7	3.55

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[1] K. SHURE and O. J. WALLACE, *Nucl. Sci. Eng.*, **62**, 736 (1977).

[2] C. M. EISENHAUER and G. L. SIMMONS, *Nucl. Sci. Eng.*, **56**, 263 (1975).

[3] A. R. VETTER and A. B. CHILTON, *Nucl. Technol.*, **11**, 268 (1971).

[4] The computations were carried out by T. A. Keys, with the use of the University of Illinois IBM 360/75 computer. The results were kindly checked by K. Shure.

[5] C. M. EISENHAUER, Personal Communication (1977).