Letters to the Editor

Comments on "The Concept of Spatial Channel Theory Applied to Reactor Shielding Analysis"

The recent paper by Williams and Engle[1] is welcomed, since it gives a concise mathematical formulation and impressive sample demonstrations of widely known methods that have escaped such unifying documentation until now. The groundwork for the theory was laid by Case et al.[2] as early as 1953, with their equivalence theorem, allowing the reduction of any spatially finite problem to an infinite-medium problem. This theorem with its corollaries[2] is implicitly used by the authors in the above paper to formulate the theory. However, concluding their theory section (Sec. II), Williams and Engle make the following somewhat vague and misleading statements:

> . . . Eq. (8) can be thought of as the transport equation for the contributon flux:
>
> $$\nabla \cdot \boldsymbol{D}(\boldsymbol{r}) = S(\boldsymbol{r}) \ , \tag{24}$$
>
> where
>
> $\boldsymbol{D}(\boldsymbol{r})$ = contributon current given by Eq. (15)
>
> $$S(\boldsymbol{r}) = \int_E \int_\Omega [\phi^+(\boldsymbol{\rho}), Q(\boldsymbol{\rho}) - \phi(\boldsymbol{\rho}), Q^+(\boldsymbol{\rho})] d\Omega dE$$
>
> $$= \text{source of contributons.} \tag{25}$$
>
> Written in this form, the point reciprocity equation is merely the continuity equation for contributons. It differs from the Boltzmann transport equation by the absence of loss terms due to material interactions. This should be expected since contributons are never absorbed, and scattering events cannot be seen due to the integration over angle and energy.

These statements can lead the reader to believe that Eq. (24) is the basic equation from which the spatial distribution of the contributon current $\boldsymbol{D}(\boldsymbol{r})$ can be calculated. However, at close inspection, it is noticed that the source term $S(\boldsymbol{r})$, defined in Eq. (25), contains the phase-space distributions of the forward and adjoint fluxes, $\phi(\boldsymbol{\rho})$ and $\phi^+(\boldsymbol{\rho})$. To obtain these latter distributions, one must solve the forward and adjoint transport equations, and that solution, therefore, is a prerequisite to solving the above Eq. (24). However, once $\phi(\boldsymbol{\rho})$ and $\phi^+(\boldsymbol{\rho})$ are known, it seems much easier to calculate $\boldsymbol{D}(\boldsymbol{r})$ from its defining equation:

$$\boldsymbol{D}(\boldsymbol{r}) = \int_E \int_\Omega \boldsymbol{\Omega}\phi(\boldsymbol{\rho})\,\phi^+(\boldsymbol{\rho}) d\Omega dE \ , \tag{15}$$

than by performing the integral of Eq. (25) and then solving Eq. (24). From these reasons, Eq. (24) *is not* a "transport equation for the contributon flux"; it is just the continuity equation for contributons in an implicit form.

In the final two sentences of their theory section, Williams and Engle emphasize that Eq. (24) does not contain any loss terms due to material interactions because contributons are never absorbed. Such argumentation can be misleading because the fact that contributons can never be lost in the process of streaming from source to detector does not mean that their distribution within the transporting medium is not influenced by the properties of this medium! In fact, we have, in a parallel development to the Williams and Engle paper, developed two equations for the contributon flux[3] that can be considered the real transport equations for contributons or "ψ-particles," as we named the product $\psi = \phi\phi^+$. For a purely absorbing medium, the monoenergetic transport equation for contributons in slab geometry has the form[3]

$$\mu^4\left(\frac{\partial^2\psi}{\partial x^2}\right)^2 + 4\mu^2 QR\,\frac{\partial^2\psi}{\partial x^2} - \mu^2\Sigma^2\left(\frac{\partial\psi}{\partial x}\right)^2 - 4\Sigma^2 QR\psi + 4Q^2R^2 = 0 \ , \tag{A}$$

where $R \equiv Q^+$ in the Williams and Engle notation and Σ is the absorption cross section of the medium ($\mu = \cos\phi$, as conventionally used). Reference 3 gives also a second form of a transport equation for ψ when isotropic scattering is allowed.

The above equation, Eq. (A), is a "real transport equation for contributons" because it allows the calculation of ψ from first principles and does *not* require the solution of either the forward or adjoint form of the Boltzmann equation. The nonlinearity of Eq. (A) and other unusual features are further discussed in Ref. 3.

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[1] M. L. WILLIAMS and W. W. ENGLE, Jr., *Nucl. Sci. Eng.*, **62**, 92 (1977).

[2] K. M. CASE, F. deHOFFMANN, and G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, Vol. I, Chap. V, Los Alamos Scientific Laboratory (1953).

[3] S. A. W. GERSTL, "A New Concept for Deep-Penetration Transport Calculations and Two New Forms of the Neutron Transport Equation," LA-6628-MS, Los Alamos Scientific Laboratory (1976).

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In reply to Gerstl's[1] comments concerning spatial channel theory,[2] the authors feel that his concern over referring to the contributon continuity equation [Eq. (24) in Ref. 2] as the "contributon transport equation" is more an argument over semantics than substance. Equation (24)

[1] S. A. W. GERSTL, *Nucl. Sci. Eng.*, **64**, 798 (1977).

[2] M. L. WILLIAMS and W. W. ENGLE, Jr., *Nucl. Sci. Eng.*, **62**, 92 (1977).