The Relations Between Various *Contributon* Variables Used in Spatial Channel Theory

Chilton's remarks[1] concerning spatial channel theory point out the apparent lack of rigor in treating *contributon* flux rather than *contributon* current as the basic channel theory quantity. His comments are welcome as well as enlightening.

Since spatial channel theory was introduced for application to shielding analysis, there has been some discussion concerning the significance of *contributon* flux versus *contributon* current. The authors of Ref. 2, themselves, seem to have perpetrated some of these questions through their use of the phrase ". . . it is intuitively obvious that . . . ." We would like to add to Chilton's comments some other perhaps enlightening thoughts that were considered "intuitively obvious" in the previous development.

The basic channel theory equation can be written for nonsource, nondetection regions as

$$\nabla \cdot \boldsymbol{D}(r) = 0 \quad , \tag{1}$$

where

$$\boldsymbol{D} = \textit{contributon current}$$

$$= \int_E \int_\Omega \boldsymbol{v} n(r, E, \Omega)\, \phi^*(r, E, \Omega)\, d\Omega dE \quad .$$

By defining the mean *contributon* velocity to be

$$\bar{\boldsymbol{v}}_c = \frac{\iint \boldsymbol{v} n \phi^* d\Omega dE}{\iint n\phi^* d\Omega dE} \quad , \tag{2}$$

and the *contributon* density to be

$$\rho_c(r) = \iint n\phi^* d\Omega dE \quad , \tag{3}$$

Eq. (1) becomes

$$\nabla \cdot \bar{\boldsymbol{v}}_c \rho_c(r) = 0 \quad . \tag{4}$$

Equation (4) is identical to the conservation-of-mass equation of fluid mechanics. Since $\rho_c(r)$ has units of "response per unit volume," $\rho_c(r)dV$ must be at least proportional to the potential response contained in dV, just as ρdV represents the mass of fluid in volume dV.

The major motivation for considering *contributon* flux rather than *contributon* density or current is that engineers are accustomed to dealing with a flux variable. To understand the significance of *contributon* flux, we define the mean *contributon* speed as

$$\bar{v}_c = \overline{|\boldsymbol{v}|} = \frac{\int_E \int_\Omega |\boldsymbol{v}| n\phi^* d\Omega dE}{\int_E \int_\Omega n\phi^* d\Omega dE} = \frac{C(r)}{\rho_c(r)} \quad , \tag{5}$$

where $C(r)$ is the *contributon* flux. In general,

$$\overline{|\boldsymbol{v}|} \neq |\bar{\boldsymbol{v}}| \quad ,$$

i.e., the mean *contributon* speed is not equal to the magnitude of the mean *contributon* velocity.

During a time interval, dt, the *contributons* located at $\boldsymbol{r}$ will travel a total tracklength distance of

$$\bar{v}_c \rho_c dt \quad .$$

Therefore, if we consider an infinitesimal sphere centered at r, and if all the *contributons* had the average speed $\bar{v}_c$, then $C(r)$ would represent the rate per unit volume that *contributons* at r flow through the sphere.

The relation between $C(r)$ and $\boldsymbol{D}(r)$ is found by examining $\bar{\boldsymbol{v}}_c$ and $\bar{v}_c$. The mean velocity is a vector in the same direction $\boldsymbol{e}_D$ as $\boldsymbol{D}$ and with some magnitude $|\bar{\boldsymbol{v}}_c|$. Thus, Eq. (1) can be written as

$$\nabla \cdot \hat{e}_D |\bar{\boldsymbol{v}}_c| \rho_c = 0 \tag{6}$$

or

$$\nabla \cdot \hat{e}_D \frac{|\bar{\boldsymbol{v}}_c|}{\overline{|\boldsymbol{v}|}_c} C(r) = 0 \quad . \tag{7}$$

Comparing Eqs. (1) and (7), the following relation is obtained:

$$|\boldsymbol{D}(r)| = \frac{|\bar{\boldsymbol{v}}_c|}{\overline{|\boldsymbol{v}|}_c} C(r) \quad .$$

In most shielding analysis, streaming occurs predominately in one direction, and in this case,

$$|\bar{\boldsymbol{v}}_c| \simeq \overline{|\boldsymbol{v}|}_c \quad ,$$

so that

$$|\boldsymbol{D}| \simeq C(r) \quad .$$

There are instances, however, in which *contributons* exist at r, but have $|\bar{\boldsymbol{v}}_c| \simeq 0$. An example of such a case was given in Ref. 2. In this situation, $|\boldsymbol{D}| \sim 0$, while $C(r)$ may possibly be quite large. For this reason, we have chosen to use the quantity $C(r)$ in most channel theory analysis, although as Chilton points out, $\boldsymbol{D}(r)$ can be a more useful variable for some cases.

The philosophical question as to whether *contributon* flux, *contributon* current, or *contributon* density is the most "basic" quantity is as meaningless as discussing whether neutron flux is more fundamental than neutron current. All *contributon* variables can be useful; their relative merit depends on the particular situation at hand.

It is perhaps most convenient to view channel theory as a mathematical transformation (analogous, for example, to a Laplace transformation) from the particle domain into a "response domain." Mathematically, this transformation can be applied to any quantity defined in terms of the particle density: the particle flux, current, lifetime, reaction rates, etc. All have corresponding values in the response domain. An interesting example of analysis in the response domain is given by familiar perturbation theory, which can be viewed as finding the change in the *contributon* population due to changes in reactor properties. It may well be that other response variables besides those discussed in this Letter have useful practical application, and further investigation of the physical characteristics of the *contributon* variables is certainly warranted.

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[1]A. B. CHILTON, *Nucl. Sci. Eng.*, **63**, 219 (1977).
[2]M. L. WILLIAMS and W. W. ENGLE, Jr., *Nucl. Sci. Eng.*, **62**, 92 (1977).