Rejoinder to "Comments on 'Systems with Stochastic Parameters' "

Williams[1] has made some observations concerning the recent paper of Karmeshu and Bansal[2] dealing with the calculations of first and second moments of a simple neutronic system that is stochastically perturbed. In this analysis we have assumed that the stochastic reactivity is a dichotomic Markov process (DMP) and the source is a white noise (delta correlated) Gaussian process. It should be emphasized that these calculations are exact.

Williams[1] makes the assertion that the results obtained by us[2] for the first moments can be obtained directly from Sec. 8 of his paper.[3] However, he has mentioned in this section that it is not possible to obtain an exact solution for the case when delayed neutrons are present, and therefore only an approximate technique can be applied. Williams' solution is not exact because of the approximations to the closure property, namely,

$$\langle \Delta(t) N(t) \rangle \simeq \langle \Delta(t) \rangle \langle N(t) \rangle$$

and

$$\langle \Delta(t_1)\, \Delta(t_2) N(t_2) \rangle \simeq \langle \Delta(t_1)\, \Delta(t_2) \rangle \langle N(t_2) \rangle \ .$$

Here $\Delta(t)$ is the stochastic perturbation in the reactivity and is assumed to be stationary Gaussian. It should be stressed that in the DMP the closure property holds exactly.[4] Thus, if $\Delta(t)$ is a DMP and $\phi[\Delta(\ldots)]$ is a functional of this process involving only times prior to t_1, then for $t > t_1$,

$$\langle \Delta(t)\, \Delta(t_1)\, \phi[\Delta(\ldots)] \rangle = \langle \Delta(t)\, \Delta(t_1) \rangle \langle \phi[\Delta(\ldots)] \rangle \ . \tag{1}$$

Because of this, the moments obtained by Karmeshu and Bansal[2] are exact. Williams' assertion that our technique for calculating the first moment has no material advantage, except conciseness, over the iteration technique discussed by Bourret[5,6] does not appear to be tenable, since the technique we employed, besides affording the advantage of conciseness, leads to exact expressions for the moments. More recently Bourret et al.[4] have studied stochastic stability of a linear differential equation with stochastic parameter characterizing a DMP. Bourret et al. assert in Sec. 8 of their paper: "This is the first time that all second-order properties of a linear stochastic differential equation are obtained explicitly; although it is assumed that the stochastic perturbation is a DMP." Karmeshu and Bansal[2] have also studied a physical system, namely, the simple neutronic system described by a stochastic differential equation with the stochastic parameter characterizing a DMP. Our analysis closely follows that of Bourret et al.[4] and hence the expressions for moments are exact. Thus exact calculation of second moment in our paper is an additional advantage over that of Williams.[3] Moreover, our calculations[2] correspond to finite correlation time of the stochastic parameter, while the corresponding expressions for the white noise stochastic parameter are obtained for infinitesimally short correlation time.

Recently Akcasu and Karasulu[7] have studied the statistical properties of the power response of point reactor to a stochastic reactivity insertion and a stochastic source of external neutrons, assuming the reactivity and source variations to be white noise processes.

The equation of the mean obtained by them is at variance with the conclusions of Williams.[8] Akcasu and Karasulu attributed this discrepancy to the neglect of some terms in the evaluation of the coefficients occurring in the Fokker-Planck equation. They further mention that the approximation

$$\int_t^{t+\delta t} \Delta(t') X(t')\, dt' \simeq \left[\int_t^{t+\delta t} \Delta(t')\, dt'\right] X(t) \tag{2}$$

employed by Williams is not valid for Gaussian white noise; this approximation as pointed out by Leibowitz[9] seems to have appeared at least implicitly in a number of places.

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