In the writer's opinion, the answer is given in Ref. 4, Secs. 3 and 4, regardless of whether the interstitial ("moderator") region is dealt with by diffusion or transport theory.

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Comments on "Systems with Stochastic Parameters"

In a recent paper, Karmeshu and Bansal[1] have considered the response of the point model reactor kinetics equations to a random parametric reactivity excitation. They make the statement that their results differ from those of Williams[2] because he assumed that no correlation exists between reactivity and neutron density. It is the purpose of this Letter to point out the reason for the neglect of this correlation in Williams' paper and to emphasize the importance of this particular problem.

The reasoning outlined here follows closely that of Gray and Caughey.[3] We consider the point model reactor kinetics equations with one group of delayed neutrons in the form

$$\frac{dN(t)}{dt} = \frac{1}{l}[\rho(t) - \beta]N(t) + \lambda C(t) + S(t) \tag{1}$$

$$\frac{dC(t)}{dt} = -\lambda C(t) + \frac{\beta}{l} N(t) \tag{2}$$

$$\rho(t) = \rho_0 + \Delta(t) \quad \text{and} \quad S(t) = S_0 + \mathcal{J}(t) \quad .$$

Now $\Delta(t)$ and $\mathcal{J}(t)$ can be considered either as Gaussian, physical white noise sources or, alternatively, as derivatives of a Wiener process such that

$$\Delta(t) = \frac{\delta W}{\delta t} \quad \text{and} \quad \mathcal{J}(t) = \frac{\delta V}{\delta t} \quad , \tag{3}$$

where δW and δV are jump processes. Equations (1) and (2) with the excitation given by Eq. (3) can be interpreted in a discrete sense by assuming that, at the start of each time interval, δt, the system receives a random impulse that sends it from state P_1 to P_2 instantaneously due to the action of the term $(\delta WN + \delta V)/l$. From this stage the system moves according to the term $[(\rho_0 - \beta)N/l + \lambda C + S_0]\delta t$ until the end of the time interval when it is a state P_3. The process is then repeated indefinitely, with impulses followed by steady motion in successive time intervals δt. This type of behavior is characteristic of Brownian motion, shot noise, or neutron emission from fission. In this case, N and δW are uncorrelated.

The other situation arises when δW and δV are a continuous process such that $\delta W/\delta t$ is a mathematical approximation for a Gaussian process with a very short correlation time. In this case N and δW are correlated.

Thus, when we formulate the Fokker-Planck equation and are required to evaluate the limit

$$A_1 = \lim_{\delta t \to 0} \frac{\langle \delta N \rangle}{\delta t} \quad , \tag{4}$$

then for Wiener impulses we obtain

$$A_1 = \frac{1}{l}(\rho_0 - \beta)N + \lambda C + S_0 \quad , \tag{5}$$

whereas for the Gaussian continuous process we obtain

$$A_1 = \frac{1}{l}\left(\rho_0 - \beta + \frac{\sigma_{11}^2}{l}\right) + \lambda C + S_0 \quad , \tag{6}$$

where

$$\langle \delta W \rangle = 0 \tag{7}$$

and

$$\langle (\delta W)^2 \rangle = 2\sigma_{11}^2 \, \delta t \quad . \tag{8}$$

When these and the other coefficients are inserted into the Fokker-Planck equation, it can be readily shown[4] that the corresponding first moment equations are:

Wiener Process

$$\frac{d\langle N \rangle}{dt} = \frac{1}{l}(\rho_0 - \beta)\langle N \rangle + \lambda \langle C \rangle + S_0 \tag{9}$$

Gaussian Process

$$\frac{d\langle N \rangle}{dt} = \frac{1}{l}\left(\rho_0 - \beta + \frac{\sigma_{11}^2}{l}\right)\langle N \rangle + \lambda \langle C \rangle + S_0 \quad . \tag{10}$$

Williams[2] obtained Eq. (9), and Karmeshu and Bansal have obtained the solution of Eq. (10) where $D = \rho_0^2\sigma_{11}^2$ and not $D = \sigma_{11}^2$ as suggested. It might also be mentioned that the solution of Eq. (10) was obtained by Williams[5] by the renormalization technique [see Eq. (8.10) of that paper].

Now we must discuss which equation is the correct one. There is no doubt that, if we wish $\Delta(t)$ to simulate the effects of random neutron injection from fission, Eq. (9) must be correct. This follows from the fact that Eq. (9) is the exact moment equation of the zero power probability balance equation. On the other hand, when the reactivity perturbation is due to random vibration or some other *mechanical* cause, it seems physically more realistic to adopt the Gaussian assumption and employ Eq. (10). From the practical point of view it does seem that an experimental investigation of these two types of stochastic perturbation would be useful. Initial indications from the work of Akcasu[6] on boiling water reactors suggest that its effect is not insignificant.

Finally, it must be pointed out that the technique described by Karmeshu and Bansal for calculating the first moments has no material advantage, other than conciseness over the iteration technique discussed by Bourret[7,8] in his earlier works. In addition, all the results quoted by Karmeshu and Bansal for the first moments can be obtained directly from Sec. 8 of the paper by Williams.[5]

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