Letter to the Editor

Remarks on "Neutron Transport with Temperature Feedback"

We have recently shown that a neutron transport problem in a finite homogeneous body with temperature feedback leads to the following nonlinear abstract differential system in a suitable Banach space X,

$$du(t)/dt = (B + J)\,u(t) + F[u(t)] + v_0 \ , \quad t < 0 \ ;$$

$$\lim_{t\to 0+} u(t) = u_0 \ , \quad (1)$$

where $(B + J)$ is a generalized Boltzmann integro-differential operator and $F[u]$ is a nonlinear operator due to temperature feedback.[1]

System (1) was transformed into the nonlinear abstract integral equation

$$u(t) = \left[Z(t)\,u_0 + \int_0^t Z(t-s)\,v_0\,ds\right] + \int_0^t Z(t-s)\left\{Ju(s) + F[u(s)]\right\} ds \ , \quad (2)$$

where $Z(t) = \exp(tB)$ is the semigroup generated by B.

We proved that the integral Eq. (2) admits a unique strongly continuous solution $u = u(t)$, defined at any $t \in [0,\bar{t}]$, provided that $\bar{t}$ is suitably chosen. We then observed that it was difficult to decide whether or not such an $u(t)$ satisfied the differential system, Eq. (1).

We want here to remark that, due to the special form of $F[u]$ [see Eq. (11) of Ref. 1], $u(t)$ is also a strong solution of the differential system, Eq. (1). This can be verified directly as follows. Define

$$R(t,h) = \bar{u}(t) - [u(t+h) - u(t)]/h \ ,$$

where $\bar{u}(t)$ satisfies the linear integral equation *formally* obtained from Eq. (2) by substituting $s' = t - s$ and then by differentiating with respect to t. By a few manipulations, it can then be proven that $R(t,h)$ satisfies a linear integral equation with an arbitrarily small known term. Hence, $\lim R(t,h) = 0$ as $h \to 0$, uniformly for $t \in [0,\bar{t}]$. This shows that $u(t)$ is differentiable and that $\bar{u} = du/dt$. Finally, by using standard techniques, it is easy to prove that $u(t)$ satisfies Eq. (1), provided that $u_0 \in D(B)$ (Ref. 2). This result agrees with Theorem 7 of Ref. 3 and with Theorem 2 of Ref. 4.

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