Letters to the Editor

Comments on "Analysis of the Microfission Reactor Concept"

Cole and Renken[1] have recently questioned the feasibility of utilizing laser- or electron-beam irradiation of pelletized fissionable material to produce fission microexplosions that, on a continuing basis, could become a commercial source of fission energy. I will try to answer those questions.

THE COMPRESSION ENERGY

Here I show briefly how I arrived at the approximation $E \simeq (2/3)pV$.

The energy for adiabatic compression is given by $E = pV/(\gamma - 1)$, where $\gamma = c_p/c_v$ is the specific heat ratio. For a monatomic gas and for a degenerate electron gas one has to put $\gamma = 5/3$ giving $E = (3/2)pV$. From experimental data of high explosives[2] one gets a rather good fit by putting $\gamma \approx 3$. The value of $\gamma \approx 3$ may be a better approximation at the beginning of the compression process and the value $\gamma = 5/3$ ($\sim$2) a better approximation at the end of the compression process. Assuming an average value of $\gamma = 2.5$, one obtains $E = pV/(\gamma - 1) = (2/3)pV$. The value of the compression energy for plutonium given in Ref. 1 is $0.89pV$, near my estimate of $0.67pV$.

REFLECTED PELLETS

The conclusions of Ref. 1 assume a cold reflector surrounding the fissionable core. In case the reflector is hot ($\sim$10 keV), the neutron thermal velocity is $\simeq 1.4 \times 10^8$ cm/sec. The number of collisions the neutron experiences before reentering the core is estimated as follows. The neutron-scattering mean-free-path in a medium of atomic density $N \simeq 5 \times 10^{26}$ cm^{-3} and a scattering cross section $\sigma_s \simeq 2 \times 10^{-24}$ cm^2 is $\lambda \simeq 10^{-3}$ cm. After n scatterings, the neutron has traveled $\lambda\sqrt{n}$ cm. The neutron reenters the core if this distance is the same order as the core radius $R_{min} \simeq 5 \times 10^{-3}$ cm, so that $n = 25$. The total path length of the neutron before reentering the core is $l = n\lambda = 2.5 \times 10^{-2}$ cm. At a speed of $v = 1.4 \times 10^8$ cm/sec, the time to traverse this path is $\tau = l/v \simeq 2 \times 10^{-10}$ sec, shorter than the pellet disassembly time. In case the pellet consists of several concentric layers of fissionable and neutron scattering material or a mixture of them, the time for the neutrons to reenter the core will be considerably shorter.

FISSION-FUSION PELLETS

The microfission concept aids the ignition of a thermonuclear reaction provided the neutron reflecting or scattering substance is a thermonuclear material such as TD or ^{6}LiD mixed with the fission material (see, for example, Ref. 3).

If the heat released by the fission chain reaction ignites a thermonuclear reaction in the T-D envelope, a large number of fusion neutrons are produced, accelerating the pace of the chain reaction, releasing more heat, and further raising the temperature in the thermonuclear material whereby more fusion neutrons are produced. Therefore, both the fission chain reaction and the thermonuclear fusion reaction are coupled in a bootstrap mode, such that the fusion reaction is supported by a fission chain reaction and the thermonuclear fusion reaction is supported by a fission chain reaction. As a result of this coupling, the critical mass of a fissionable pellet surrounded by a fusionable envelope at thermonuclear temperatures is reduced. Since a pellet, consisting of both fissionable and fusionable material, can be compressed and heated to thermonuclear temperatures by a laser-, electron-, or ion-beam pulse, the reduction in the critical mass as a function of the temperature becomes an interesting quantity.

We estimate this effect by considering a homogeneous mixture of fissionable (^{233}U, ^{235}U, ^{239}Pu) and fusionable (T-D) material at a pressure of $p = 10^{18}$ dyn/cm^2. The atomic number densities would be $N_u = 1.17 \times 10^{25}$ cm^{-3} and $N_h = 5 \times 10^{26}$ cm^{-3}, respectively. Introducing the mixing parameter x, $0 < x < 1$, defined such that in the mixture of fissionable and fusionable material there are $(1 - x)N_u$ fissionable and xN_h fusionable atoms per cm^3, the neutron chain reaction for such a mixture of infinite extension is described by

$$\frac{1}{v_0}\frac{\partial \phi}{\partial t} = (\nu - 1)(1 - x)N_u \sigma_f \phi + S , \qquad (1)$$

where

v_0 = neutron velocity

ϕ = neutron flux

ν = number of neutrons produced per fission

σ_f = neutron fission cross section

S = neutron source arising in the T-D fusion reaction.

Assuming for simplicity that the neutron velocities of the fission and fusion neutrons are the same, which to order of magnitude is true, the source term can be written as

$$S = \tfrac{1}{4}\, x^2 N_h^2 \overline{\sigma v} , \qquad (2)$$

[1] RANDALL K. COLE and JAMES H. RENKEN, *Nucl. Sci. Eng.*, **58**, 345 (1975).

[2] R. SCHALL, "Detonation Physics," in *Physics of High Energy Density*, p. 234, Academic Press, New York (1971).

[3] F. WINTERBERG, in *Laser Interaction and Related Plasma Phenomena*, Vol. 3, p. 519 ff, Plenum Press, New York (1974).

where $\overline{\sigma v}$ is the velocity-averaged fusion cross section of the T-D reaction. In the interesting temperature range from 1 to 10 keV the following approximation, with T in keV, can be applied:

$$\overline{\sigma v} \simeq 1.1 \times 10^{-20}\, T^{4.37} \quad . \tag{3}$$

After inserting into Eq. (1), one has

$$\frac{1}{v_0}\frac{\partial \phi}{\partial t} = (\nu - 1)(1 - x)N_u\sigma_f\phi + 2.75 \times 10^{-21} x^2 N_h^2 T^{4.37} \quad . \tag{4}$$

A relation between the temperature, T, and the neutron flux is obtained by taking into account the heat released by the rapidly growing nuclear reactions resulting from both the fission and the fusion processes. Part of this released heat becomes particle kinetic energy of the background plasma and part becomes black-body radiation, the latter because the opacity of the plasma, partly constituted by fissionable material, is very high. The particle kinetic energy is of the order NkT and the black-body radiation energy of the order aT^4, where k is the Boltzmann constant and a is the Stefan-Boltzmann constant. For $N \approx 10^{26}$ cm^{-3} (corresponding to a highly compressed state) and $T \approx 1$ keV, one has $NkT \approx 10^{17}$ erg/cm^3 and $aT^4 \approx 10^{14}$ erg/cm^3. At $T \approx 10$ keV both particle kinetic energy and black-body radiation energy would be of the same order of magnitude. In the temperature range of interest below 10 keV one can, therefore, neglect the black-body radiation energy as compared to the particle kinetic energy.

For an atomic number density, N_h, of 5×10^{26} cm^{-3} and a neutron cross section, σ_s, of 2×10^{-24} cm^2, the neutron mean-free-path is 10^{-3} cm, which is smaller than the critical radius. The neutron kinetic energy is, therefore, dissipated into the plasma rather than lost by unattenuated neutron escape. In addition, the charged fission and ^{4}He fusion products, which have a much smaller range, almost completely dissipate their kinetic energy into the plasma environment.

The fission power per unit volume is given by

$$\epsilon_f(1 - x)N_u\sigma_f\phi \quad ,$$

where ϵ_f(=180 MeV) is the fission reaction energy, and the T-D fusion power per unit volume is given by

$$\epsilon_\alpha S = \epsilon_\alpha x^2 N_h^2 \overline{\sigma v}/4 = 2.75 \times 10^{-21}\, \epsilon_\alpha x^2 N_h^2 T^{4.37} \quad ,$$

where ϵ_α(=17.2 MeV) is the fusion reaction energy.

With these heat sources the temperature increase in the fission-fusion plasma follows from

$$3k\,[g(1 - x)N_u + xN_h]\,\frac{\partial T}{\partial t} = \epsilon_f(1 - x)N_u\sigma_f\phi + 2.75 \times 10^{-21}\, \epsilon_\alpha x^2 N_h^2 T^{4.37} \quad , \tag{5}$$

where $k = 1.6 \times 10^{-9}$ erg/keV is the Boltzmann constant and g is the degree of ionization of the fissionable material such that $1 < g < 93$. A reasonable estimate of g is 10.

Expanding the function $f(T) = T^{4.37}$ around $T = T_0$(>1 keV) into a Taylor series, one has

$$T^{4.37} = \text{const} + 4.37\, T_0^{3.37}\, T \quad . \tag{6}$$

By inserting this expansion into Eqs. (4) and (5), one obtains the following set of coupled linear differential equations:

$$\frac{\partial \phi}{\partial t} = \alpha_1\phi + \beta_1 T + \gamma_1 \tag{7}$$

$$\frac{\partial T}{\partial t} = \alpha_2\phi + \beta_2 T + \gamma_2 \quad , \tag{8}$$

where

$$\alpha_1 \equiv (\nu - 1)(1 - x)N_u\sigma_f v_0$$

$$\beta_1 \equiv 1.2 \times 10^{-20} v_0 x^2 N_h^2 T_0^{3.37}$$

$$\alpha_2 \equiv \frac{\epsilon_f(1 - x)N_u\sigma_f}{3k\,[g(1 - x)N_u + xN_h]}$$

$$\beta_2 \equiv \frac{1.2 \times 10^{-20}\, \epsilon_\alpha x^2 N_h^2 T_0^{3.37}}{3k\,[g(1 - x)N_u + xN_h]} \quad .$$

Furthermore, γ_1, γ_2 are constants, the values of which are of no interest. Equations (7) and (8) can be brought into one single differential equation for either ϕ or T. The differential equation for T is

$$\ddot{T} - (\alpha_1 + \beta_2)\dot{T} + (\alpha_1\beta_2 - \alpha_2\beta_1)T + \alpha_1\gamma_2 - \alpha_2\gamma_1 = 0 \quad . \tag{9}$$

The general solution of Eq. (9) is the sum of a particular solution of the inhomogeneous equation and the general solution of the homogeneous equation. A particular integral of the inhomogeneous equation into which the constants γ_1 and γ_2 enter has the form T = const and is of no interest. The homogeneous equation,

$$\ddot{T} - (\alpha_1 + \beta_2)\dot{T} + (\alpha_1\beta_2 - \alpha_2\beta_1)T = 0 \quad , \tag{10}$$

has the general solution

$$T = \text{const}\,\exp(\lambda t) \quad , \tag{11}$$

with

$$\lambda = \frac{\alpha_1 + \beta_2}{2} + \left[\left(\frac{\alpha_1 + \beta_2}{2}\right)^2 + \alpha_2\beta_1 - \alpha_1\beta_2\right]^{1/2} \quad , \tag{12}$$

for the physically meaningful solution increasing with time.

For $x = 0$, corresponding to a pure fission plasma,

$$\lambda = \lambda_0 = (\nu - 1)N_u\sigma_f v_0 \quad , \tag{13}$$

which is the same growth rate as that of a pure fission chain reaction in a pure fission plasma. For a pure fission chain reaction in a fission-fusion plasma, but where the fission chain reaction is assumed not to be coupled to the fusion neutrons,

$$\lambda_1 = \alpha_1 = \lambda_0(1 - x) \quad . \tag{14}$$

In reality, however, the fission chain reaction is coupled to the fusion process and the value of λ given by Eq. (12) is larger than the value of λ_1 given by Eq. (14).

For $0 < x < 1$ a factor,

$$f = \lambda/\lambda_0 \quad , \tag{15}$$

gives the change in the rate of the fission-fusion chain reaction compared to a pure fission chain reaction in purely fissionable material. From this factor, f, an effective neutron multiplication factor, ν^*, can be deduced by putting

$$\nu^* - 1 = (\nu - 1)f \quad . \tag{16}$$

It is convenient to introduce the auxiliary function, $F(x)$, defined by

$$F(x) = \frac{x^2}{g + \frac{x}{1 - x}\frac{N_h}{N_u}}\,\frac{N_h^2}{N_u^2} \quad , \tag{17}$$

so that

$$\alpha_2 = 2.1 \times 10^8\, \epsilon_f\sigma_f(N_u/N_h)^2 F(x)/x^2 \tag{18}$$

and

$$\beta_2 = 2.5 \times 10^{-12}\, \epsilon_\alpha T_0^{3.37} N_u F(x)/(1 - x) \quad . \tag{19}$$

One can now easily show that for $x \approx 0.5$ and for 1 keV < T < 10 keV (with other numerical parameters as given before), one has

$$\alpha_1 \beta_2 \ll \alpha_2 \beta_1 \quad , \tag{20}$$

and

$$\left(\frac{\alpha_1 + \beta_2}{2}\right)^2 \ll \alpha_2 \beta_1 \quad . \tag{21}$$

It therefore follows that

$$\lambda \gtrsim (\alpha_2 \beta_1)^{1/2} \quad . \tag{22}$$

From Eqs. (4) and (5) one can see that this approximation is equivalent to neglecting the fission neutron source term in Eq. (4), it being small in comparison to the fusion neutron source term, and neglecting the fusion reaction heat source term in Eq. (5) in comparison to the fission reaction heat source term.

With the definition of f [Eq. (15)] one has from Eq. (22)

$$f(x) \gtrsim 4.26 \left[\frac{x^2(1-x)}{1+3.3x}\right]^{1/2} T_0^{1.68} \quad . \tag{23}$$

This function has a maximum for $x = 0.57$ where

$$f \gtrsim 0.94\, T_0^{1.68} \quad . \tag{24}$$

Assume, for example, that T_0 = 5 keV; then f = 14.2 and ν^* = 28.0. For T_0 = 10 keV, f = 45.0 and ν^* = 86.5.

With this computed value for f, one can estimate the reduction in the required critical mass due to the bootstrap coupling of the fission chain reaction with the fusion process.

Since the critical radius is proportional to $(\nu - 1)^{-1/2}$, it is reduced by the factor $f^{1/2}$ and the critical mass reduced by the factor $f^{3/2}$. For f = 45 the critical mass is reduced by ≃300. To attain a large f-value, the pellet must be heated from $T = 0$ to $T \approx 10$ keV, which requires an energy approximately 10 times the compression energy, such that an overall reduction in the energy of ~30 is required to achieve criticality.

The effect of the fission-fusion bootstrap on the exponential growth of the chain reaction is even more pronounced. The exponential growth of the chain reaction is determined by the Rossi-α, which is proportional to $(\nu - 1)$. In the case of the fission-fusion chain reaction, Rossi-α has to be multiplied by the factor f (=45). Since the relative yield is determined by the factor $R\alpha$, where R is the pellet radius, an increase in α by a factor 45 would permit an equal reduction in R to achieve the same relative yield. This would imply a reduction in the pellet volume by a factor $(45)^3$ ($\sim 10^5$). Reference 1 gives a value for the compression energy of $\sim 2 \times 10^8$ J required to achieve a substantial yield. This compression energy could thus be reduced by $\sim 10^5$. Simultaneous heating from $T = 0$ to $T = 10$ keV requires ~10 times more energy so that an overall reduction in energy by $\sim 10^4$ (that is, from 2×10^8 to 2×10^4 J) can be achieved.

In addition to the coupling effect given here, there is an even more direct coupling as the fast-moving fission products kick off fusionable nuclei to attain kinetic energies required to overcome the fusion barrier. This effect becomes increasingly more important with higher pellet densities.

OTHER ASPECTS OF IMPORTANCE

Although the fission-fusion hybrid pellets can require as much trigger energy as pure fusion pellets, the character of the rapidly growing fission-fusion chain reaction leads to high burnup yield for both nuclear components. Since magnetohydrodynamic systems can convert the energy of the fireball into useful energy with a much higher efficiency then in conventional fission reactors, a much better utilization of the fissionable fuel is achieved. Furthermore, should the limited efficiency of laser trigger systems or the pulse shaping problems of relativistic electron beams pose a serious problem, as would be the case for laser- or e-beam fusion, the use of bunched-ion beams may become a very interesting alternative.[4]

F. Winterberg

Desert Research Institute
University of Nevada System
Reno, Nevada 89507

June 25, 1975

[4]F. WINTERBERG, *Plasma Phys.*, **17**, 69 (1975).

Reply to "Comments on 'Analysis of the Microfission Reactor Concept' "

THE COMPRESSION ENERGY

The relation $E = (2/3)pV$, quoted without explanation, gives the impression of being an exact result. (Note that it is "=" rather than "≃" in a majority of Winterberg's papers.) The only potentially applicable exact model is the degenerate electron gas for which $E = (3/2)pV$, and we assumed (as did all other workers with whom we discussed it) that this was the intended model. Of course, the difference is really not very significant. Our Thomas-Fermi-corrected calculation (the best simple model available) is theoretical over-kill, and the difference of 34% in energy for plutonium (or even the factor-of-2 differences for reflector materials) is small compared to other uncertainties in the calculation—or to the several orders of magnitude by which the whole scheme misses practicality.

REFLECTED PELLETS

The discussion here misses the essential point. The fact that Winterberg's estimated time for a reflected neutron to return to the fissionable core, $\sim 2 \times 10^{-10}$ sec, is shorter than the inertial confinement time is irrelevant. It is much *greater* than the e-folding time of the neutron population as a whole, given by the inverse of Rossi-α, which must be $\sim 3 \times 10^{-11}$ sec for explosive yields from pellets of the size considered here. Reflected neutrons simply arrive too late to have much effect on the diverging chain.

FISSION-FUSION PELLETS

While interesting, Winterberg's comments have no relation to our paper, which considered pure fission only. Without making detailed calculations (a far-from-trivial undertaking), it is impossible for us to do more than suggest that the argument presented is dangerously simplified, and the conclusion counter to our intuition.

R. K. Cole, Jr.
J. H. Renken

Theoretical Division 5223
Sandia Laboratories
Albuquerque, New Mexico 87115

July 14, 1975