

The generalized functions are defined as follows:

$$(\mathbf{A}_0^+ - \lambda_0 \mathbf{B}_0^+) \Gamma^+ = \frac{(\delta \mathbf{A}^+ - \lambda_0 \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\delta \mathbf{A} - \lambda_0 \delta \mathbf{B}) \phi_0 \rangle} - \frac{(\mathbf{B}_0^+ + \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\mathbf{B}_0 + \delta \mathbf{B}) \phi_0 \rangle}, \quad (9)$$

$$(\mathbf{A}_0^+ - \lambda_0 \mathbf{B}_0^+) \Gamma_\lambda^+ = \frac{(\delta \mathbf{A}^+ - \lambda_0 \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\delta \mathbf{A} - \lambda_0 \delta \mathbf{B}) \phi_0 \rangle} - \frac{\mathbf{B}^+ \phi_0^+}{\langle \phi_0^+, \mathbf{B} \phi_0 \rangle}, \quad (10)$$

and

$$\left(-\frac{\alpha_0}{v} + \mathbf{A}_0^+ - \mathbf{B}_{p0}^+\right) \Gamma_\alpha^+ = \frac{(\delta \mathbf{A}^+ - \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\delta \mathbf{A} - \delta \mathbf{B}) \phi_{\alpha 0} \rangle} - \frac{\mathbf{B}^+ \phi_0^+}{\langle \phi_0^+, \mathbf{B} \phi_{\alpha 0} \rangle}. \quad (11)$$

The perturbation operators $\delta \mathbf{A}$ and $\delta \mathbf{B}$ pertain to the actual alterations in the reactor, whereas $\delta \bar{\mathbf{A}}$ and $\delta \bar{\mathbf{B}}$ take into account those alterations that result also from criticality reset. The function \mathbf{B}_p is that part of the fission operator that takes into account the contribution of the prompt fission neutrons. It is concluded that the Stacey and UG versions are but two of many versions of GPT.

Applicability of Different Versions of GPT

The preceding discussion indicates that there is no generally preferred version of GPT. Each has its own range of applicability. The Stacey version is the right formulation for calculation of the effect of alterations on integral parameters that are functions of the static eigenvalue. Hence, it is not surprising that the Stacey formulation yields the static reactivity more accurately⁵ than does the UG version. Similarly, the α -reset version of GPT is expected to be more accurate for calculating the effect of system alterations on such integral parameters as the prompt-mode reactivity and decay constant. Many system alterations encountered in the design and operation of nuclear reactors maintain criticality. For example, the change in the fuel composition due to burnup is compensated by a change in the concentration of burnable poisons. Uncertainties in input cross sections must be compensated in the design by changes in the composition or geometry of the reactor. The mechanism used to restore criticality can contribute significantly¹³⁻¹⁵ to the effect of the alteration on different integral parameters. The UG version of GPT is the appropriate version for assessing the effect of those physical alterations that leave the reactor critical.

Terminology

It might be useful if a unified terminology were established for what is becoming an important field of perturbation theory. I propose that the term generalized perturbation theory be used for all perturbation-theory formulations in which the flux and adjoint perturbations are allowed for as correction factors that make first-order expressions correct to the second order. There are different versions of GPT, and these can be classified according to two categories: (a) the approach of allowing for the flux and adjoint perturbations, and (b) for homogeneous systems, the criticality-reset mechanism.

The perturbations in the distribution functions can be taken into account either in terms of generalized functions or in terms of perturbations in distribution functions.

¹³A. GANDINI, M. SALVATORES, and I. DAL BONO, in *Fast Reactor Physics*, Vol. I, p. 241, International Atomic Energy Agency, Vienna (1968).

¹⁴A. GANDINI and M. SALVATORES, *Nucl. Sci. Eng.*, **41**, 452 (1970).

¹⁵H. H. HUMMEL and W. M. STACEY, Jr., *Nucl. Sci. Eng.*, **54**, 35 (1974).

Equations (1), (2), and (3) provide examples for the generalized-function formulation. The same equations can also be expressed in terms of the flux alteration:

$$\rho_{\lambda I} = \rho_0 \left\{ 1 + \left\langle \delta \phi, \left[\frac{(\delta \mathbf{A}^+ - \lambda_0 \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\delta \mathbf{A} - \lambda_0 \delta \mathbf{B}) \phi_0 \rangle} - \frac{\mathbf{B}^+ \phi_0^+}{\langle \phi_0^+, \mathbf{B} \phi_0 \rangle} \right] \right\rangle \right\}, \quad (12)$$

$$\rho_{\lambda V} = \rho_0 \left\{ 1 + \left\langle \delta \phi_\lambda, \left[\frac{(\delta \mathbf{A}^+ - \lambda_0 \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\delta \mathbf{A} - \lambda_0 \delta \mathbf{B}) \phi_0 \rangle} - \frac{\mathbf{B}^+ \phi_0^+}{\langle \phi_0^+, \mathbf{B} \phi_0 \rangle} \right] \right\rangle \right\}, \quad (13)$$

and

$$\rho_\alpha = \rho_{\alpha 0} \left\{ 1 + \left\langle \delta \phi_\alpha, \left[\frac{(\delta \mathbf{A}^+ - \delta \mathbf{B}^+) \phi_0^+}{\langle \phi_0^+, (\delta \mathbf{A}^+ - \delta \mathbf{B}^+) \phi_{\alpha 0} \rangle} - \frac{\mathbf{B}^+ \phi_0^+}{\langle \phi_0^+, \mathbf{B} \phi_{\alpha 0} \rangle} \right] \right\rangle \right\}, \quad (14)$$

where

$$(\mathbf{A}_0 - \lambda_0 \mathbf{B}_0) \delta \phi = -(\delta \bar{\mathbf{A}} - \lambda_0 \delta \bar{\mathbf{B}}) \phi, \quad (15)$$

$$(\mathbf{A}_0 - \lambda_0 \mathbf{B}_0) \delta \phi_\lambda = -[\delta \mathbf{A} - \delta(\lambda \mathbf{B})] \phi_\lambda, \quad (16)$$

and

$$\left(-\frac{\alpha_0}{v} + \mathbf{A}_0 - \mathbf{B}_{p0}\right) \delta \phi_\alpha = -\left(-\frac{\delta \alpha}{v} + \delta \mathbf{A} - \delta \mathbf{B}_p\right) \phi_\alpha. \quad (17)$$

In general, the generalized-function formulation is useful⁹ for problems requiring the calculation of the effect of different system alterations on a given integral parameter. Conversely, the distribution alteration is the useful approach for problems requiring the calculation of the effect of a given system alteration on different integral parameters.

Each of the GPT formulations should also be classified according to the criticality-reset mechanism. For example, Eq. (13) is referred to, in the terminology proposed, as the k -reset version of GPT for the static reactivity expressed in terms of the flux alteration. Similarly Eq. (3) is the α -reset version of GPT for the prompt-mode reactivity expressed in terms of generalized functions.

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The Streaming Term of the Transport Equation in Terms of General Orthogonal Coordinates

From time to time, papers appear that suggest that the evaluation of the streaming term in the transport equation is a complicated and laborious process when the coordinate system is not Cartesian. (See, for example, Ref. 1.) In fact, it is easy to do the calculation in a compact manner. Perhaps everyone knows the scheme I shall describe. However, although I have used it for some time in teaching, I know of no reference in which it is easily available. Perhaps, for this reason, I may be excused for presenting what might be common knowledge.

¹G. C. POMRANING and C. A. STEVENS, *Nucl. Sci. Eng.*, **55**, 359 (1974).

Consider a system of orthogonal coordinates denoted (q_1, q_2, q_3) or, simply, (q) . Let

$$\frac{\partial}{\partial q_i} \mathbf{r}(q) = h_i(q) \hat{e}_i(q) \tag{1}$$

and

$$\frac{\partial}{\partial q_i} \hat{e}_i = \frac{1}{h_i} \sum_{k=1}^3 \Gamma_k^{ij}(q) \hat{e}_k(q) \tag{2}$$

Then,

$$\Gamma_k^{ij} = \frac{1}{2h_k} \left(\frac{\partial}{\partial q_i} g_{jk} - \frac{\partial}{\partial q_k} g_{ji} \right) \tag{3}$$

with $g_{ij} = \delta_{ij} h_i^2$. The \hat{e}_i forms the local base system; the h_i gives the differential of length $d\bar{s}^2 = \sum_i h_i^2 d\bar{q}_i^2$. The three-index symbols Γ_k^{ij} are defined in a manner slightly different from the Christoffel symbols of tensor analysis.² Their expression in terms of h_i^2 is easily derived. [For example, compare $\frac{\partial^2}{\partial q_i \partial q_j} \mathbf{r}$ with $\frac{\partial^2}{\partial q_j \partial q_i} \mathbf{r}$. Then, note that $\frac{\partial}{\partial q_j} (\hat{e}_i \cdot \hat{e}_k) = 0$.]

Once Eqs. (1), (2), and (3) are accepted, the streaming term can be evaluated effortlessly. We wish to express $\mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} f(\mathbf{r}, \mathbf{v})$ (where the bar reminds that \mathbf{v} is to be held constant) in terms of derivatives $\frac{\partial}{\partial q_i}$, all $v_j \equiv (\mathbf{v} \cdot \hat{e}_j)$ held constant, and derivatives $\frac{\partial}{\partial v_i}$, all q_j held constant. We then have

$$\mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} f(\mathbf{r}, \mathbf{v}) = \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} f_1[q, \mathbf{v} \cdot \hat{e}_1(q), \mathbf{v} \cdot \hat{e}_2(q), \mathbf{v} \cdot \hat{e}_3(q)] \tag{4}$$

$$= \frac{v_i}{h_i} \left(\frac{\partial}{\partial q_i} + \Gamma_k^{ij} \frac{v_k}{h_j} \frac{\partial}{\partial v_j} \right) f_1(q, v) \tag{5}$$

$$= \frac{v_i}{h_i} \frac{\partial}{\partial q_i} f_1 + \frac{v_k}{h_k} \left(v_k \frac{\partial}{\partial q_i} h_k - v_i \frac{\partial}{\partial q_k} h_i \right) \frac{1}{h_i} \frac{\partial}{\partial v_i} f_1 \tag{6}$$

and that is the end of the calculation. [We use the summation convention in Eqs. (5) and (6).]

As an example, we evaluate Eq. (6) for the torus discussed by Pomraning and Stevens.¹ The coordinates are similar to those of the right circular cylinder. One has a pair of plane polar coordinates $q_2 = \rho$, $q_3 = \theta$, $h_2 = 1$, and $h_3 = \rho$ and a coordinate $q_1 = \theta_1$ (rather than $q_1 = z$), which locates the circular section. Corresponding to q_1 is $h_1 = R + \rho \sin \theta \equiv \rho_1$, where R is the radius of the axis of the torus. Then, Eq. (6) becomes

$$\begin{aligned} \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} f &= \frac{v_1}{\rho_1} \frac{\partial}{\partial \theta_1} f_1 + v_2 \frac{\partial}{\partial \rho} f_1 + \frac{v_3}{\rho} \frac{\partial}{\partial \theta} f_1 \\ &- \frac{v_1}{\rho_1} (v_2 \sin \theta + v_3 \cos \theta) \frac{\partial}{\partial v_1} f_1 + \left(\frac{v_1^2}{\rho_1} \sin \theta + \frac{v_3^2}{\rho} \right) \frac{\partial}{\partial v_2} f_1 \\ &+ \left(\frac{v_1^2}{\rho_1} \cos \theta - \frac{v_2 v_3}{\rho} \right) \frac{\partial}{\partial v_3} f_1 \end{aligned} \tag{7}$$

The transition to the right circular cylinder is achieved by setting $\frac{1}{\rho_1} \frac{\partial}{\partial \theta_1} = \frac{\partial}{\partial z}$ in the first group of terms and neglecting all terms containing ρ_1 in the second group.

An interesting special case occurs when the speed of the particle is fixed. Then, one of the three components of velocity can be eliminated. For example, introduce the variables (v, η, ξ) through $v_1 = v \cos \eta$, $v_2 = v \sin \eta \cos \xi$, and $v_3 = v \sin \eta \sin \xi$. Then, the second group of terms in Eq. (7) becomes ($v = 1$)

$$\begin{aligned} &\frac{\cos \eta}{\rho_1} \sin(\theta + \xi) \frac{\partial}{\partial \eta} f_2(q, \eta, \xi) \\ &+ \left[\frac{\cot \eta}{\rho_1} \cos \eta \cos(\theta + \xi) - \frac{\sin \eta}{\rho} \sin \xi \right] \frac{\partial}{\partial \xi} f_2(q, \eta, \xi) \end{aligned}$$

These should be compared with Eq. (30) of Ref. 1, after a typographical error has been corrected.

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²See, for example, E. MADELUNG, *Die Mathematischen Hilfsmittel des Physikers*, Dover Publications, Inc., New York (1943).

Corrigendum

M. MARTINI, G. PALMIOTTI, and M. SALVATORES, "A Benchmark Experiment of Neutron Propagation in Iron Used to Test ENDF/B Cross-Section Data," *Nucl. Sci. Eng.*, **56**, 427 (1975).

The second sentence of the Conclusions should read as follows:

The results so far obtained show good agreement between calculation and experiment when the ENDF/B-I data or the more recent data based on an ORNL evaluation (MAT 4180 Mod. 1) are used with proper accounting of the manganese-impurity background effect.