Letters to the Editors

A Comparison of the Transfer Function of the Internal Reactor Loop for the Cases of both Distributed and Lumped Parameters

The thermal processes in the reactor core are described by partial differential equations; in stability considerations, some simplifications must be introduced to permit simulation of these equations on an analogue computer. This is usually done by replacing the distributed parameters by lumped ones.

In a previous communication (*1*) the time response of the coolant temperature has been considered. The influence of the same simplifying assumptions as in (*1*), but on the dynamic behavior of the internal reactor loop (Fig. 1) is the scope of this letter.

The presented analysis is performed for the single fuel channel with the following assumptions:

1. The reactor is nearly critical and the reactivity change as a disturbing value, is less than one "dollar."

2. The coolant and the moderator are incompressible media.

3. The fuel element without cladding is of the rod type, with no conduction in the longitudinal direction, and with a radially symmetric temperature distribution. The heat source distribution along this fuel element is represented by

$$q(z,t) = w(t) q_0 \sin \frac{\pi}{L} z \tag{1}$$

4. The neutron density n and coolant temperature T are considered as deviations from the steady state.

5. The inlet coolant temperature is constant.

6. The zero power kinetic transfer function $K_R G_R(s)$ is derived from the normal set of neutron kinetics equations.

The frequency response of the coolant temperature is calculated for the following three models: (1) The parameters of the fuel-coolant channel are treated as lumped ones and the thermal conductivity of the fuel element is assumed to be $\lambda = \infty$. (2) The parameters are treated as distributed and $\lambda = \infty$. (3) The parameters are distributed but $\lambda \neq \infty$. The solutions of the equations which describe the transient heat transfer of the coolant channel are presented in ref. *1*. The power coolant temperature transfer functions obtained from these solutions are for the above-mentioned three models respectively

$$1.\quad G_k(s) = \frac{1}{(1+\tau_1 s)(1+\tau_2 s)} \sum_{i=1}^{k} \frac{1}{2} \left[-\cos \frac{\pi}{L} z \right]_{z_i}^{z_{i+1}} \cdot \exp \left\{ -\frac{1}{w} \left[L - \frac{1}{2} L_x - \frac{(i-1)}{k} L \right] \right\} \tag{2}$$

$$2.\quad G_D(s) = \frac{q_0 u k' (\pi/L)}{2 w^2 F \gamma c} \cdot \frac{s + (uk'/c_f \gamma_f F_f)}{[A(s)]^2 + (\pi^2/L^{-2})[s + (uk'/c_f \gamma_f F_f)]^2} [1 + e^{-A(s)L}] \tag{3}$$

$$3.\quad G_\lambda(s) = \frac{q_0 u \alpha \pi}{2 L c_f \gamma_f F w \gamma c} \frac{1}{[B(s)]^2 + (\pi^2/L^2)} \frac{2}{a^2} \cdot \sum_i \frac{\xi_i^2}{(\alpha/\lambda)^2 + \xi_i^2} \frac{a}{\xi_i} \frac{J_1(\xi_i a)}{J_0(\xi_i a)} \times \frac{1}{s + (\lambda \xi_i^2 / c_f \gamma_f)} [1 + e^{-B(s)L}] \tag{4}$$

where

$$(1+\tau_1 s)(1+\tau_2 s) = \frac{c_f \gamma_f V_f}{2uk'w} s^2 + \left(\frac{L}{2w} + \frac{c_f \gamma_f V_f}{2F\gamma wc} + \frac{c_f \gamma_f V_f}{uLk'} \right) s + 1 \tag{5}$$

$$A(s) = \left(\frac{1}{w} s + \frac{uk'}{F\gamma wc} \right) \left(s + \frac{uk'}{c_f \gamma_f F_f} \right) - \frac{(uk')^2}{c_f \gamma_f F_f F \gamma wc} \tag{6}$$

$$B(s) = \frac{1}{w} s + \frac{u\alpha}{Fw\gamma c} - \frac{u\alpha^2}{Fw\gamma c c_f \gamma_f} \frac{2}{a} \sum_i \frac{\xi_i^2}{(\alpha/\lambda)^2 + \xi_i^2} \frac{1}{s + (\lambda \xi_i^2 / c_f \gamma_f)} \tag{7}$$

and ξ_i are roots of the equation

$$\xi_i J_0'(\xi_i a) + \frac{\alpha}{\lambda} J_0(\xi_i a) = 0 \tag{8}$$

where

a	=	outer fuel element radius
c, c_f	=	coolant and fuel specific heat respectively
F, F_f	=	area of channel and fuel cross section respectively
k	=	number of sections
k'	=	fuel element to coolant heat transfer coefficient
L	=	fuel element length
L_x	=	length of fuel element section
u	=	perimeter of fuel element
V_f	=	volume of fuel element
w	=	coolant velocity
z	=	space variable along the fuel element axis
z_i	=	coordinates which define the division points of the fuel channel
α	=	heat transfer coefficient
γ, γ_f	=	coolant and fuel specific gravity respectively

The numerical examples are used to compare three models of the internal reactor loop. The main data of these examples are: fuel element of the rod type of 0.013 meter diameter;

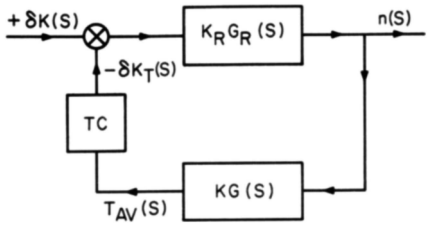

FIG. 1. Reactor internal feedback loop

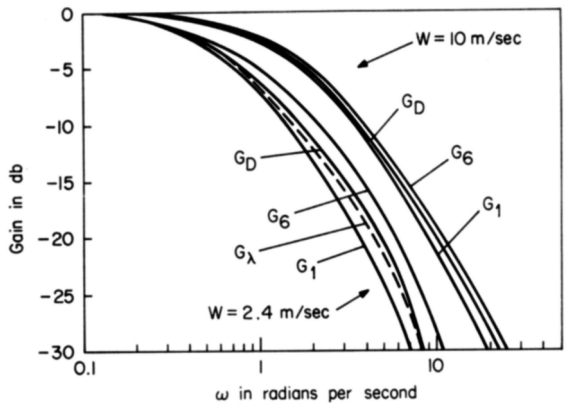

FIG. 2. Attenuation characteristics of the power-coolant temperature transfer functions

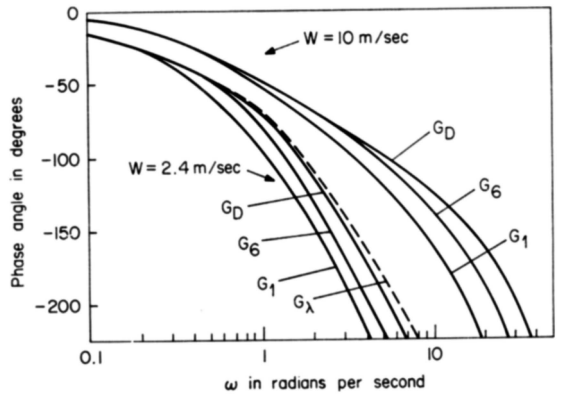

FIG. 3. Phase angle characteristics of the power-coolant temperature transfer functions

$F = 0.88 \times 10^{-4}$ meter; $L = 1.5$ meters. The power-coolant transfer functions are considered for two values of the coolant velocity, that is, $w = 2.4$ meters/sec and 10 meters/sec, and the results as attenuation and phase characteristics are presented in Figs. 2 and 3 respectively. After adding the attenuation and phase characteristics of the power-coolant temperature and zero power kinetics transfer functions, the open-loop frequency responses of the examples considered are obtained, and shown in Figs. 4 and 5 respectively. This is done when the zero power kinetics transfer function is considered for two values of $l^* = 5 \times 10^{-5}$ sec and 5×10^{-4} sec.

In order to compare the dynamic behavior of the loop considered when the coolant system is described by the

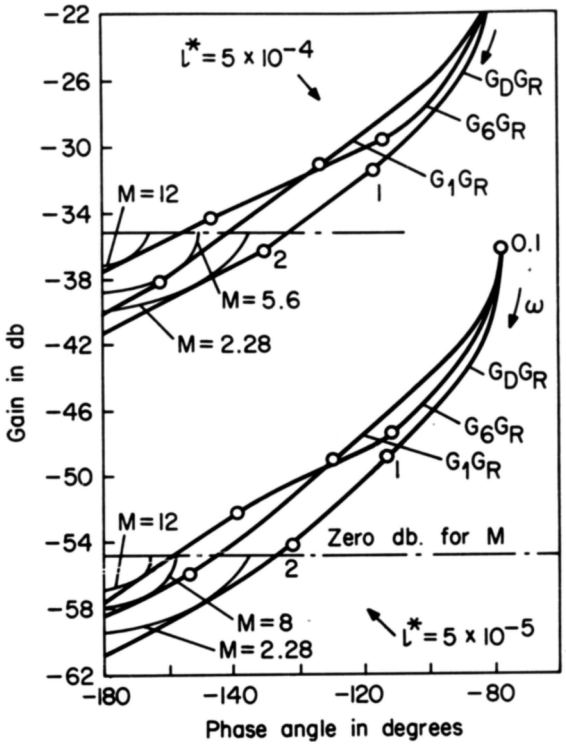

FIG. 4. Open-loop transfer functions of reactor internal loop ($w = 2.4$ meters/sec)

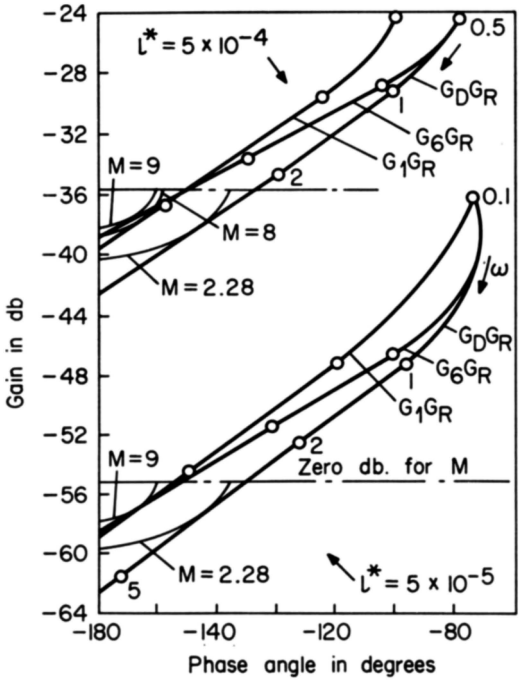

FIG. 5. Open loop transfer functions of reactor internal loop ($w = 10$ meters/sec)

distributed and lumped parameters, we adjusted the open loop gain $KTC\ K_R$ so that the maximum gain M_p of the closed loop for the distributed representation is equal to $M_p = 2.28$ db. Then it is evident from Figs. 4 and 5 that the gain of the closed loop of the one- and six-section models have different values depending upon the parameters w and

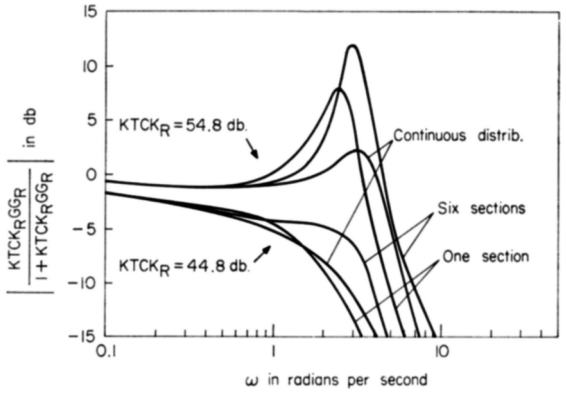

FIG. 6. Attenuation characteristics of the closed-loop transfer function $[KTCK_RGG_R/(1 + KTCK_RGG_R)]$

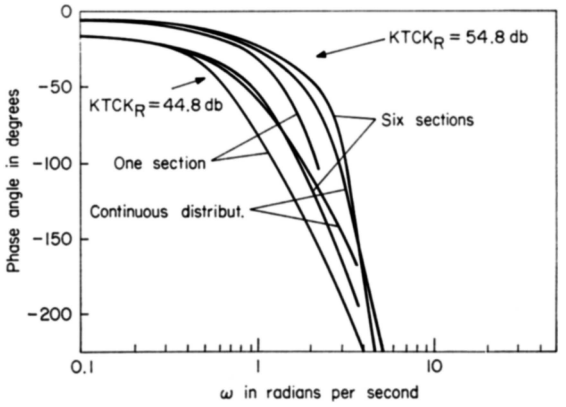

FIG. 7. Phase angle characteristics of the closed-loop transfer function $[KTCK_RGG_R/(1 + KTCK_RGG_R)]$

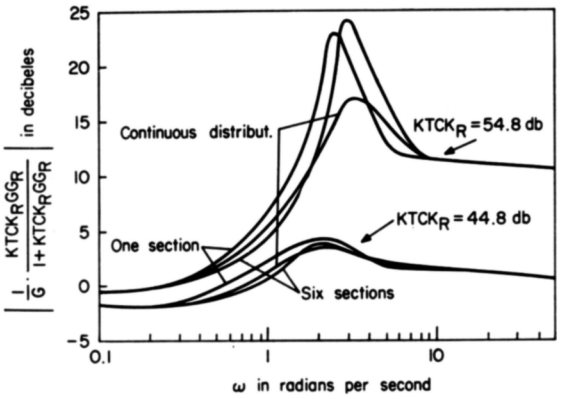

FIG. 8. Attenuation characteristics of the closed-loop transfer function $[(1/G)KTCK_RGG_R/(1 + KTCK_RGG_R)]$

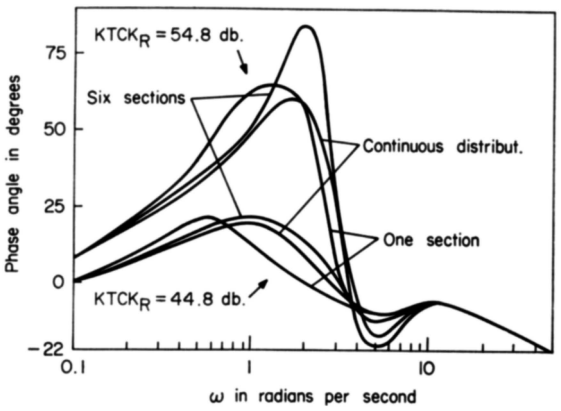

FIG. 9. Phase angle characteristics of the closed-loop transfer function $[(1/G)KTCK_RGG_R/(1 + KTCK_RGG_R)]$

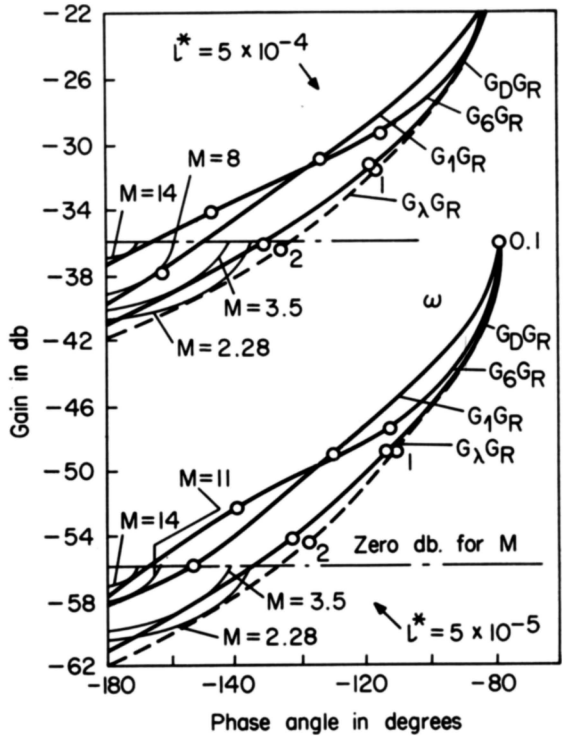

FIG. 10. Open-loop transfer functions of reactor internal feedback loop

l^*, and the differences in the resonant frequency increase as the number of sections decrease.

To show more clearly these differences, two examples of the closed loop transfer functions are considered when the zero power and power-coolant temperature transfer functions are in parallel and in series. In the first case the relation between coolant temperature and reactivity is taken as:

$$\frac{T_{AV}}{\delta k} = \frac{1}{TC}\left(\frac{KTCK_R\,GG_R}{1 + KTCK_R\,GG_R}\right) \tag{9}$$

and in the second case the output and input variables are

the neutron density and reactivity respectively

$$\frac{n}{\delta k} = \frac{1}{KTC}\left(\frac{1}{G}\frac{KTCK_R GG_R}{1 + KTCK_R GG_R}\right) \tag{10}$$

Since the terms in the brackets describe the dynamic behavior of the system, only they need be examined. Plots of the attenuation frequency and phase-shift frequency curves of these transfer functions are shown in Figs. 6–9, when the open-loop gain is equal to $KTC\,K_R = 54.8$ db and 44.8 db.

The above comparison between the distributed and lumped transfer functions was done on the basis of $\lambda \approx \infty$. To show the error that arises due to this assumption, the open-loop transfer functions for three models are plotted in Fig. 10 for the case $w = 2.4$ meters/sec. If we compare these curves with those in Fig. 4 we see that the assumption $\lambda = \infty$ makes the open-loop gain coefficient smaller than if $\lambda \neq \infty$, when in both cases the closed loop gain is equal to $M = 2.28$ db.

The results that have been presented above give a view of the significance of the simplifying assumptions made in order to simulate the effect of the distributed parameters. As shown by the numerical calculations, these assumptions cause some differences in dynamic performance of a reactor temperature feedback loop. These differences are dependent on the parameters such as coolant velocity, effective time between succeeding generations of neutrons, and open loop gain ($KTC\,K_R$). For $KTC\,K_R \leqq 44.8$, when the neutron density is a controlled variable, one section may be adequate to obtain good accuracy (Fig. 8). For $KTC\,K_R \gtrsim 54.8$, or for the case when the coolant temperature is the controlled variable, use of six reactor sections may not be satisfactory (see Figs. 6–9).

The final conclusion is that the distributed model of the thermal processes in the reactor core allows the open loop gain coefficient to be greater in comparison with the lumped model at the same conditions of damping of the closed loop. By this we mean, the lumped approximation being more instable predicts the limiting value of M_p at a lower value of the gain coefficient than does the distributed parameter system.

REFERENCES

1. W. Ciechanowicz, "Simulation of the Thermal Processes in the Reactor Core Based on an Exact Solution of the Thermal Diffusion and Heat Flow Equations," *Nucl. Sci. Eng.* **13:** 75 (1962).

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Received March 23, 1962
Revised April 30, 1963

The Fuchs–Nordheim Model with Variable Heat Capacity

In this letter we shall describe how the values of certain quantities of physical interest, such as final fuel element temperature, peak power, etc., may be computed for a pulsed reactor with variable heat capacity.

For many cases of practical importance, the heat capacity, C, of the reactor fuel elements may be assumed to vary linearly with temperature, T:

$$C = C_0 + \gamma T \tag{1}$$

where C_0 and γ are constants. The so-called Fuchs–Nordheim model (*1*) yields for the equations of motion of the reactor

$$\frac{dP}{dt} = \frac{\delta k_p - \alpha T}{l} P \tag{2}$$

$$C\frac{dT}{dt} = P \tag{3}$$

where $P(t)$ is the reactor power at time t, δk_p is the prompt reactivity insert, α is the magnitude of the prompt reactivity temperature coefficient (assumed constant), and l is the prompt neutron lifetime. In (2) and (3) delayed neutron and heat transfer effects are neglected; for narrow pulses, such as are obtained in the TRIGA reactor, these are excellent approximations. The neglect of space and neutron energy dependent effects is also a good approximation and this question will be examined elsewhere (*2*).

With the variation (1) for the heat capacity of the fuel elements the equations of motion may be integrated analytically with the result that important quantities, such as total fuel element temperature rise, peak power, and total energy release, are expressible concisely as functions of a single dimensionless parameter,

$$\sigma = \alpha C_0/\gamma(\delta k_p). \tag{4}$$

This may be seen most easily by introducing the dimensionless variables

$$x \equiv t/\tau \tag{5}$$

$$Q \equiv [\alpha l/C_0(\delta k_p)^2]P \tag{6}$$

$$\theta \equiv (\alpha/\delta k_p)T \tag{7}$$

where the asymptotic reactor relaxation time,

$$\tau = l/\delta k_p\,. \tag{8}$$

Then Eqs. (2) and (3) become

$$dQ/dx = (1 - \theta)Q \tag{9}$$

$$d\theta/dx = \sigma Q/(\theta + \sigma). \tag{10}$$

Division of (9) by (10) and integration yields for the relation between power and temperature

$$Q - Q_0 = \theta + \frac{(1 - \sigma)\theta^2}{2\sigma} - \frac{\theta^3}{3\sigma} \tag{11}$$

where Q_0 is the initial value of the power, in the above defined units. The value of the final temperature, T_∞, may be obtained to an excellent approximation by setting $Q = Q_0$ in (11), since the peak power is generally so much greater than Q_0. The useful result is

$$T_\infty = \frac{\delta k_p}{\alpha}\left[-\tfrac{3}{4}(\sigma - 1) + \tfrac{3}{4}\sqrt{(\sigma - 1)^2 + \tfrac{16}{3}\sigma}\right]. \tag{12}$$

This reduces to

$$T_\infty = 2\delta k_p/\alpha \quad \text{for} \quad \sigma = \infty \tag{13}$$