for the IBM-7090 computer. ORNL-3199 (February 1962).

3. MANFRED WÄCHTER, General Electric Company, San José, California, personal communication (September 1962).

4. M. L. TOBIAS AND T. B. FOWLER, SWAPS—An experimental program for numerical solution of the transport equation with anisotropic scattering in one-dimensional slab geometry, ORNL-TM-168 (March 1962).

5. R. BRUCE KELLOGG AND L. C. NODERER, Scaled iterations and linear equations. *J. Soc. Indust. Appl. Math.*, **8**, No. 4 (1960).

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Dancoff Correction for Several Infinitely Long Cylindrical Rings

Calculations have previously been made of the Dancoff correction for fuel rods and plates immersed in an infinite moderator (*1–5*) and for an infinitely long cylinder of moderator in fuel (*1, 5, 6*). In this letter we describe the calculation of the Dancoff correction for several infinitely long cylindrical rings of fuel and moderator.

In deriving the equations for the Dancoff correction, we have assumed that:

(i) the source density of resonance neutrons is constant in the moderator,

(ii) the fuel is black to resonance neutrons,

(iii) single collisions with moderator atoms remove resonance neutrons from the resonance energy range, and that

(iv) the lumps are infinitely long.

Using the above assumptions, Carlvik and Pershagen (*1*) have obtained the following expression for the Dancoff correction;

$$C = \frac{2}{\pi L} \int dL Ki_3(\Sigma\lambda) \cos\beta \, d\beta \tag{1}$$

where L is the fuel perimeter, Σ is the total macroscopic cross section of the moderator, Ki_3 is the Bickley function of third order (*7*), λ is a chord drawn between two points on L such that it passes through moderator only, and β is the angle between the chord and the normal to L. See Fig. 1.

Equation (1) represents the average value of

$$c = \frac{2}{\pi} \int Ki_3(\Sigma\lambda) \cos\beta \, d\beta \tag{2}$$

weighted with the perimeter L

$$C = \frac{1}{L} \int c \, dL \tag{3}$$

When the perimeter has a complex shape, it can be divided into partial perimeters, giving

$$C = \frac{1}{L} \sum_{i=1}^{n} L_i C_i, \qquad C_i = \frac{1}{L_i} \int c_i \, dL_i, \qquad L = \sum_{i=1}^{n} L_i \tag{4}$$

For the case of rings, Fig. 1, Eq. (1) gives for the inner perimeter of the moderator ring $j \neq 1, n$, of radius r_{ji},

$$C_{ji} = \frac{4}{\pi} \int_0^{\pi/2} Ki_3(\Sigma_j \lambda_{ji}) \cos\beta \, d\beta \tag{5}$$

where

$$\frac{\lambda_{ji}}{r_{j0}} = \sqrt{1 - \left(\frac{r_{ji}}{r_{j0}}\right)^2 \sin^2\beta} - \frac{r_{ji}}{r_{j0}} \cos\beta, \quad 0 \leqq \beta \leqq \pi/2 \tag{6}$$

and for its outer perimeter of radius, r_{j0},

$$C_{j0} = \frac{4}{\pi} \int_{\beta_t}^{\pi/2} Ki_3(\Sigma_j \lambda_{j0}) \cos\beta \, d\beta + \frac{4}{\pi} \int_0^{\beta_t} Ki_3(\Sigma_j \lambda_{j0}) \cos\beta \, d\beta \tag{7}$$

where $\sin\beta_t = r_{ji}/r_{j0}$, and

$$\frac{\lambda_{j0}}{r_{j0}} = \begin{cases} 2\cos\beta, & \beta_t \leqq \beta \leqq \pi/2 \\ \cos\beta - \sqrt{\left(\frac{r_{ji}}{r_{j0}}\right)^2 - \sin^2\beta}, & 0 \leqq \beta \leqq \beta_t \end{cases} \tag{8}$$

If we define

$$A(\Sigma, r_{j0}, r_{ji}/r_{j0}) \equiv \frac{4}{\pi} \int_{\beta_t}^{\pi/2} Ki_3(\Sigma_j \lambda_{j0}) \cos\beta \, d\beta \tag{9}$$

$$\begin{aligned} B(\Sigma, r_{j0}, r_{ji}/r_{j0}) &\equiv \frac{4}{\pi} \int_0^{\beta_t} Ki_3(\Sigma_j \lambda_{j0}) \cos\beta \, d\beta \\ &= \frac{r_{ji}}{r_{j0}} \cdot \frac{4}{\pi} \int_0^{\pi/2} Ki_3(\Sigma_j \lambda_{ji}) \cos\beta' \, d\beta' \end{aligned} \tag{10}$$

Eqs. (5) and (7) become

$$C_{ji} = \frac{r_{j0}}{r_{ji}} B(\Sigma_j\, r_{j0}, r_{ji}/r_{j0}) \tag{11}$$

$$C_{j0} = A(\Sigma_j r_{j0}, r_{ji}/r_{j0}) + B(\Sigma_j r_{j0}, r_{ji}/r_{j0}) \tag{12}$$

In Eq. (10), the transformation $\sin\beta = (r_{ji}/r_{j0}) \sin\beta'$ has been used.

When $j = n$, $\lambda_{ni} = \infty$ and $C_{ni} = 0$.

When $j = 1$, there are three possibilities:

(i) If the central region is fuel, Eqs. (11) and (12) apply.

(ii) If the central region is the same moderator as region 1, then $r_{ji}/r_{j0} = 0$ and $C_{10} = A(\Sigma_1 r_{10}, 0)$, having been represented by Thie (*5*).

(iii) If the central region is a void region of radius, r_{1i}, separated from the moderator of region 1 in some manner, then

$$\frac{\lambda_{10}}{r_{10}} = \begin{cases} 2\cos\beta, & \beta_t \leqq \beta \leqq \pi/2 \\ 2\left[\cos\beta - \sqrt{\left(\frac{r_{1i}}{r_{10}}\right)^2 - \sin^2\beta}\right], & 0 \leqq \beta \leqq \beta_t \end{cases} \tag{13}$$

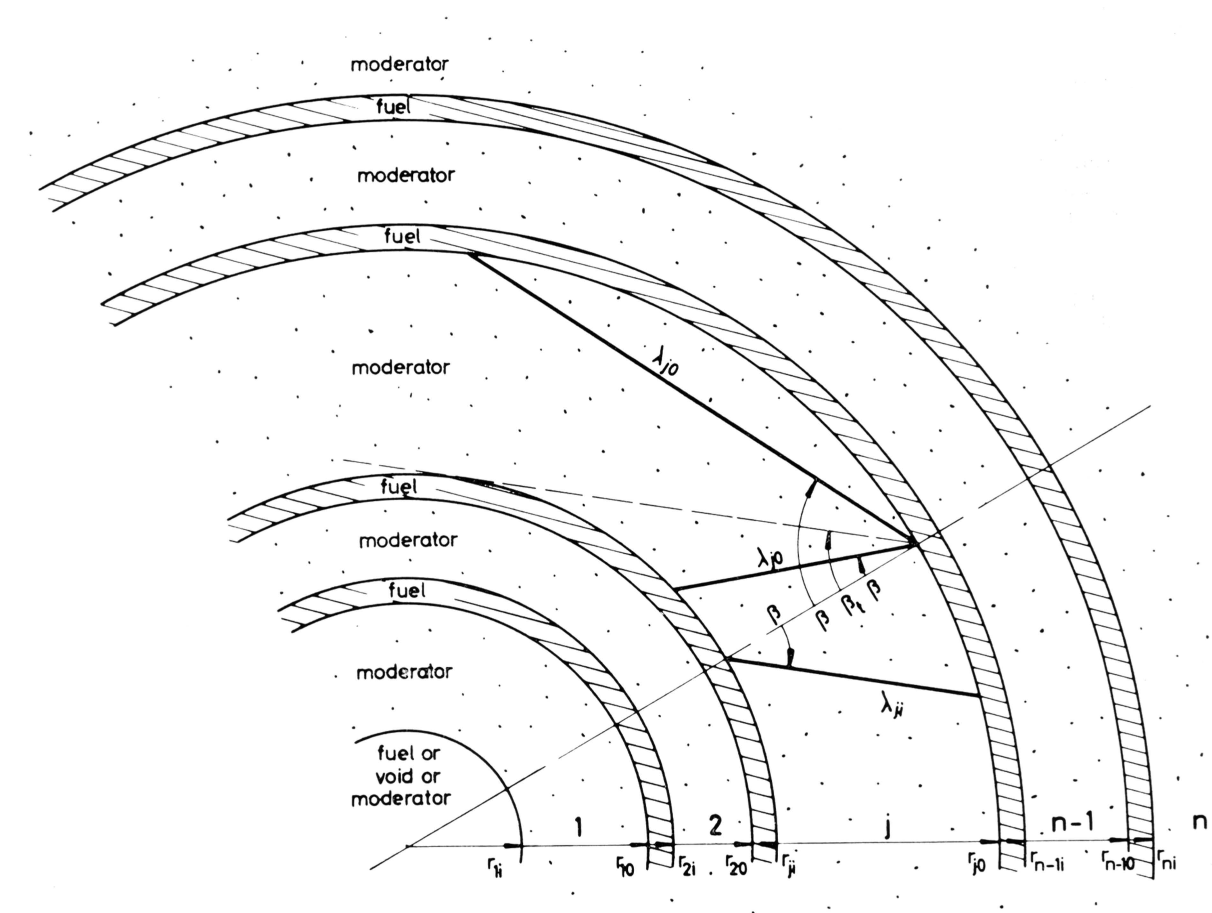

FIG. 1. Several rings of fuel and moderator

and

$$C_{10} = A(\Sigma_1 r_{10}, r_{1i}/r_{10}) + B(2\Sigma_1 r_{10}, r_{1i}/r_{10}) \quad (14)$$

Equation (14) is the same as Eq. (12) with the exception that the first argument of the function B is multiplied by 2. Equation (14) has been tabulated by Dwork (6).

The functions A and B have been numerically calculated (9) and are represented in Figs. 2 and 3. For particular values of A and B, the following apply:

$$A(\Sigma_1 r_{10}, 0) = 1 - \frac{4}{3}(\Sigma_1 r_{10})^2 \Big[2\Sigma_1 r_{10} I_0(\Sigma_1 r_{10}) K_0(\Sigma_1 r_{10}) + 2\Sigma_1 r_{10} I_1(\Sigma_1 r_{10}) K_1(\Sigma_1 r_{10}) - 2 + I_0(\Sigma_1 r_{10}) K_1(\Sigma_1 r_{10}) - I_1(\Sigma_1 r_{10}) K_0(\Sigma_1 r_{10}) + \frac{1}{\Sigma_1 r_{10}} I_1(\Sigma_1 r_{10}) K_1(\Sigma_1 r_{10}) \Big] \quad (15)$$

from (8), and

$$A(\Sigma_j r_{j0}, 1) = 0, \qquad A(0, r_{ji}/r_{j0}) = 1 - r_{ji}/r_{j0} \quad (16)$$

$$B(\Sigma_1 r_{10}, 0) = 0, \qquad B(\Sigma_j r_{j0}, 1) = 1, \qquad B(0, r_{ji}/r_{j0}) = r_{ji}/r_{j0} \quad (17)$$

When there are several rings, as in Fig. 1, the total Dancoff correction obtained using Eqs. (4), (11), and (12) is as follows:

$$C = \frac{\sum_{j=1}^{n-1} r_{j0}[A(\Sigma_j r_{j0}, r_{ji}/r_{j0}) + 2B(\Sigma_j r_{j0}, r_{ji}/r_{j0})]}{\sum_{j=1}^{n-1} (r_{ji} + r_{j0}) + r_{ni}} \quad (18)$$

when the central region is fuel,

$$C = \frac{\sum_{j=2}^{n-1} r_{j0}[A(\Sigma_j r_{j0}, r_{ji}/r_{j0}) + 2B(\Sigma_j r_{j0}, r_{ji}/r_{j0})] + r_{10} A(\Sigma_1 r_{10}, 0)}{r_{10} + \sum_{j=2}^{n-1} (r_{ji} + r_{j0}) + r_{ni}} \quad (19)$$

when the central region is the same moderator as region 1,

$$C = \frac{\sum_{j=2}^{n-1} r_{j0}[A(\Sigma_j r_{j0}, r_{ji}/r_{j0}) + 2B(\Sigma_j r_{j0}, r_{ji}/r_{j0})] + r_{10}[A(\Sigma_1 r_{10}, r_{1i}/r_{10}) + B(2\Sigma_1 r_{10}, r_{1i}/r_{10})]}{r_{10} + \sum_{j=2}^{n-1} (r_{ji} + r_{j0}) + r_{ni}} \quad (20)$$

when the central region is void.

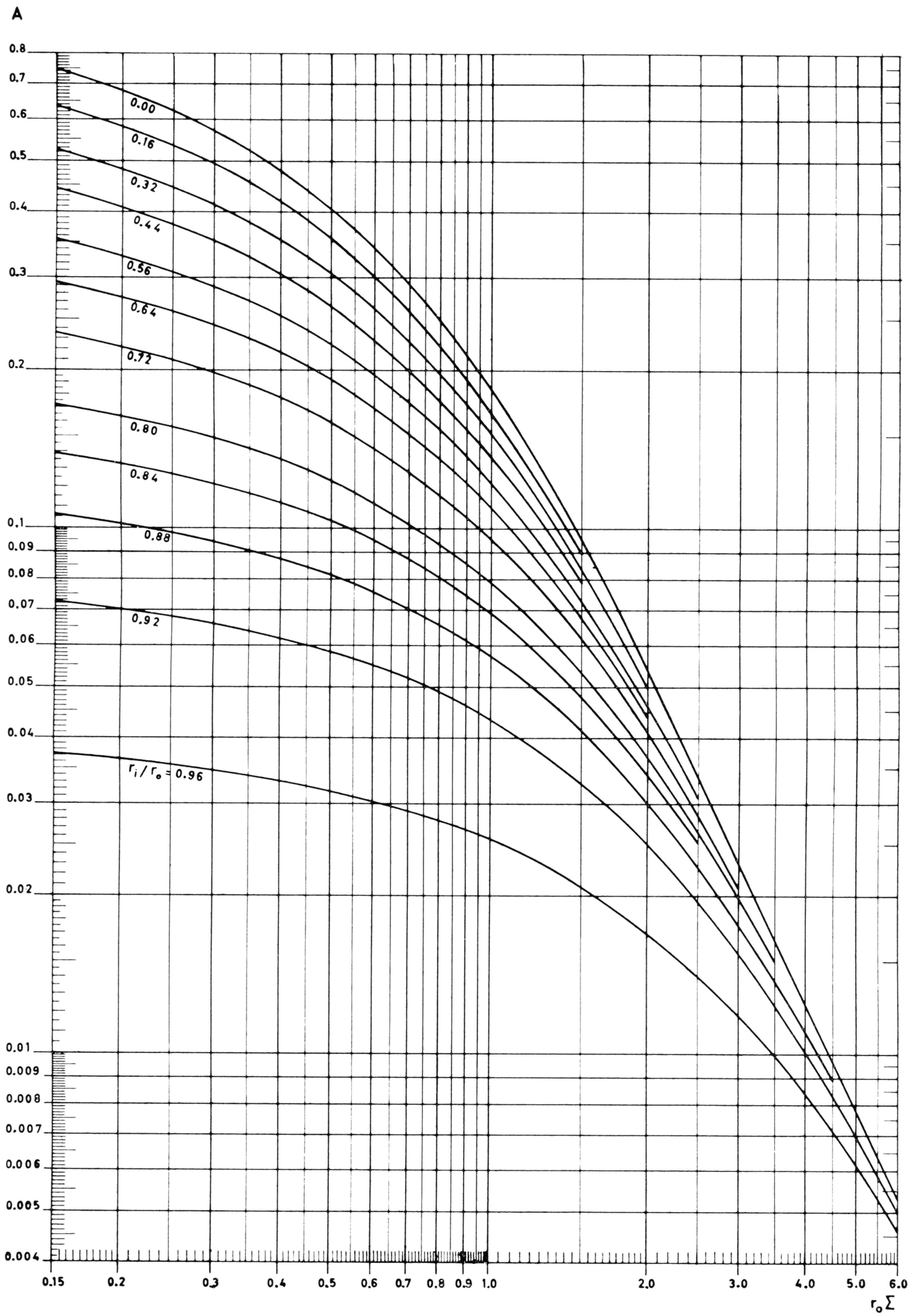

FIG. 2. Function $A(\Sigma r_0,\ r_i/r_0)$

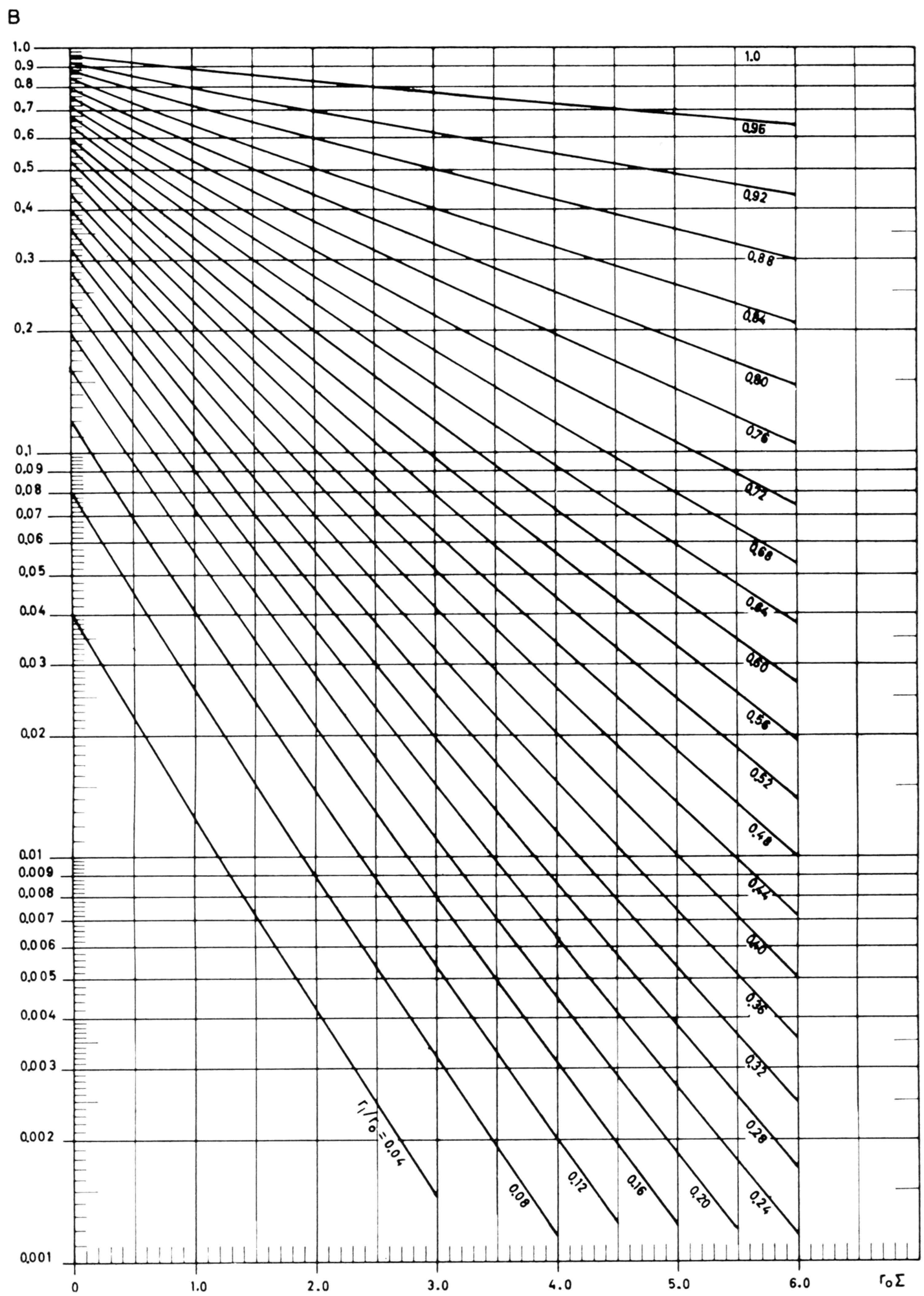

FIG. 3. Function B (Σr_0, r_i/r_0)

REFERENCES

1. I. CARLVIK AND B. PERSHAGEN, AE-16 (1959).
2. S. DANCOFF AND M. GINSBURG, CP-2157 (1944).
3. Y. FUKAI, *Nuclear Sci. and Eng.* **9**, 370–376 (1961).
4. G. VELARDE, *Rev. Energia Nuclear* **19**, 24–47 (1961).
5. J. A. THIE, *Nuclear Sci. and Eng.* **5**, 75–77 (1959).
6. J. DWORK *et al.*, KAPL-1262 (1955).
7. W. G. BRICKLEY AND J. NAYLER, *Phil. Mag.* **20**, 343 (1935).
8. K. M. CASE *et al.*, "Introduction to the Theory of Neutron Diffusion." Government Printing Office, Washington, 1953.
9. G. VELARDE, *Anales Real Soc. Españ. Fis. y Quím.* 7–9 (1961).

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