Letters to the Editors

A Note on a Simple Method for Acceleration of Finite-Difference Group-Diffusion Calculations

It is well known that cell calculations and fixed source problems are often very slow in converging in group-diffusion finite-difference programs. For example, the authors recently ran a 225 point, two-group, rectangular cell problem on the PDQ-2 (*1*) program which required about 5000 iterations[1] for a specified pointwise flux convergence; a similar number would likewise have been required in the EQUIPOISE-3 program (*2*). The equivalent case required 438 iterations on PDQ-4 (*3*). The authors have recently introduced a very simple calculational device into the EQUIPOISE-3 program which can produce a large accelerative effect. In the problem just cited the number of iterations was reduced to 70 for comparable convergence. The basic difficulty in cell calculations is that there is usually poor numerical coupling between the groups. Flux distributions within groups are readily obtained, but group-to-group flux ratios are not. The leakage terms in the finite-difference equations are usually very large compared to the slowing-down terms and the neutron balance at a point is only slightly affected by errors in the latter terms. For the whole cell, however, the leakage is zero, and the over-all neutron balance is often badly out of adjustment. It seems advisable to include some method of forcing a neutron balance between groups. In a two-group problem this is done in the following manner. It must be true at convergence that the following relations hold:

$$\text{Fast group absorptions} + \text{fast group removals} = \lambda\chi_1\,[\text{fast group productions} + \text{slow group productions}] + \text{slow group removals (upscattering)} \quad (1)$$

$$\text{Slow group absorptions} + \text{slow group removals} = \lambda\chi_2\,[\text{fast group productions} + \text{slow group productions}] + \text{fast group removals} \quad (2)$$

where λ is the reciprocal multiplication constant or eigenvalue, and χ_1 and χ_2 are the fractions of fission neutrons thrown into groups 1 and 2, respectively. At an intermediate stage in the calculation these balances do not hold. At the completion of a given iteration in EQUIPOISE-3, the eigenvalue λ and a "driving factor" D are found which will make the equations balance. The driving factor is a number by which all the fluxes in a given group are multiplied. To find λ and D, the following equations are solved simultaneously. (We assume here that D is applied to the slow fluxes.)

$$\text{Fast group absorptions} + \text{fast group removals} - D\,[\text{slow group removals}] = \lambda\chi_1\,[\text{fast group productions} + D\,(\text{slow group productions})] \quad (3)$$

$$D\,(\text{slow group absorptions}) + D\,(\text{slow group removals}) - \text{fast group removals} = \lambda\chi_2\,[\text{fast group productions} + D\,(\text{slow group productions})] \quad (4)$$

By dividing Eq. (4) by Eq. (3), λ is eliminated and D may be obtained. Having D, λ may then be computed by substituting for D in Eq. (4). The new slow fluxes are D times the old slow fluxes. In this manner, group balances are forced at each iteration, and as the calculation proceeds, the value of D approaches 1. This scheme is easily extended to a larger number of groups, but the driving factors are somewhat more difficult to obtain. Following the elimination of λ and setting one of the driving factors equal to 1, a set of simultaneous linear equations, one less than the number of groups, must be solved. As with two groups, degenerate cases can occur in which there is no numerical coupling between some of the groups and any of the others. These can usually be avoided by inspection. As a practical matter, safeguards should be built into a machine program to prevent the use of zero or infinite driving factors.

There is not much gained in using this device in noncell calculations. The authors have also experimented with similar methods in which driving factors are computed for subregions of the reactor (in a manner reminiscent of Kron's "tearing" method) but no improvement over the much simpler group relaxation scheme we have just described was obtained. We have, however, used the tearing or block relaxation procedure to advantage in transport calculations in slab geometry (*4*). In constant source problems, the driving factor would be found in a one-group problem by solving the balance equation:

$$D\,[\text{absorptions} + \text{leakages}] = \text{source} \quad (5)$$

(For several groups, the driving factors would be found by solving a set of similar equations simultaneously.) The reader is referred to a paper by Kellogg and Noderer (*5*) which discusses the mathematics of this technique in constant source problems. Use of block relaxation (tearing) rather than group relaxation may be advantageous, but we have not done any calculations to test this.

[1] Mesh sweeps or inner iterations divided by 2.

REFERENCES

1. G. G. Bilodeau, W. R. Cadwell, J. P. Dorsey, J. G. Fairey, and R. S. Varga, PDQ—An IBM-704 code to solve the two-dimensional few-group neutron-diffusion equations. Bettis Atomic Power Laboratory Report, WAPD-TM-70 (August 1957).
2. T. B. Fowler and M. L. Tobias, EQUIPOISE-3: A two-dimensional, two-group, neutron-diffusion code

for the IBM-7090 computer. ORNL-3199 (February 1962).

3. MANFRED WÄCHTER, General Electric Company, San José, California, personal communication (September 1962).
4. M. L. TOBIAS AND T. B. FOWLER, SWAPS—An experimental program for numerical solution of the transport equation with anisotropic scattering in one-dimensional slab geometry, ORNL-TM-168 (March 1962).
5. R. BRUCE KELLOGG AND L. C. NODERER, Scaled iterations and linear equations. *J. Soc. Indust. Appl. Math.*, **8**, No. 4 (1960).

MELVIN TOBIAS
DAVID R. VONDY
TOM B. FOWLER

Oak Ridge National Laboratory
Oak Ridge, Tennessee
Received August 15, 1962
Revised October 4, 1962

Dancoff Correction for Several Infinitely Long Cylindrical Rings

Calculations have previously been made of the Dancoff correction for fuel rods and plates immersed in an infinite moderator (*1–5*) and for an infinitely long cylinder of moderator in fuel (*1*, *5*, *6*). In this letter we describe the calculation of the Dancoff correction for several infinitely long cylindrical rings of fuel and moderator.

In deriving the equations for the Dancoff correction, we have assumed that:

(i) the source density of resonance neutrons is constant in the moderator,

(ii) the fuel is black to resonance neutrons,

(iii) single collisions with moderator atoms remove resonance neutrons from the resonance energy range, and that

(iv) the lumps are infinitely long.

Using the above assumptions, Carlvik and Pershagen (*1*) have obtained the following expression for the Dancoff correction;

$$C = \frac{2}{\pi L}\int dL K i_3(\Sigma\lambda)\cos\beta\, d\beta \tag{1}$$

where L is the fuel perimeter, Σ is the total macroscopic cross section of the moderator, Ki_3 is the Bickley function of third order (*7*), λ is a chord drawn between two points on L such that it passes through moderator only, and β is the angle between the chord and the normal to L. See Fig. 1.

Equation (1) represents the average value of

$$c = \frac{2}{\pi}\int Ki_3(\Sigma\lambda)\cos\beta\, d\beta \tag{2}$$

weighted with the perimeter L

$$C = \frac{1}{L}\int c\, dL \tag{3}$$

When the perimeter has a complex shape, it can be divided into partial perimeters, giving

$$C = \frac{1}{L}\sum_{i=1}^{n} L_i C_i, \qquad C_i = \frac{1}{L_i}\int c_i\, dL_i, \qquad L = \sum_{i=1}^{n} L_i \tag{4}$$

For the case of rings, Fig. 1, Eq. (1) gives for the inner perimeter of the moderator ring $j \neq 1, n$, of radius r_{ji},

$$C_{ji} = \frac{4}{\pi}\int_0^{\pi/2} Ki_3(\Sigma_j\lambda_{ji})\cos\beta\, d\beta \tag{5}$$

where

$$\frac{\lambda_{ji}}{r_{j0}} = \sqrt{1-\left(\frac{r_{ji}}{r_{j0}}\right)^2\sin^2\beta} - \frac{r_{ji}}{r_{j0}}\cos\beta, \quad 0 \leqq \beta \leqq \pi/2 \tag{6}$$

and for its outer perimeter of radius, r_{j0},

$$C_{j0} = \frac{4}{\pi}\int_{\beta_t}^{\pi/2} Ki_3(\Sigma_j\lambda_{j0})\cos\beta\, d\beta + \frac{4}{\pi}\int_0^{\beta_t} Ki_3(\Sigma_j\lambda_{j0})\cos\beta\, d\beta \tag{7}$$

where $\sin\beta_t = r_{ji}/r_{j0}$, and

$$\frac{\lambda_{j0}}{r_{j0}} = \begin{cases} 2\cos\beta, & \beta_t \leqq \beta \leqq \pi/2 \\ \cos\beta - \sqrt{\left(\dfrac{r_{ji}}{r_{j0}}\right)^2 - \sin^2\beta}, & 0 \leqq \beta \leqq \beta_t \end{cases} \tag{8}$$

If we define

$$A(\Sigma_j r_{j0}, r_{ji}/r_{j0}) \equiv \frac{4}{\pi}\int_{\beta_t}^{\pi/2} Ki_3(\Sigma_j\lambda_{j0})\cos\beta\, d\beta \tag{9}$$

$$B(\Sigma_j r_{j0}, r_{ji}/r_{j0}) \equiv \frac{4}{\pi}\int_0^{\beta_t} Ki_3(\Sigma_j\lambda_{j0})\cos\beta\, d\beta = \frac{r_{ji}}{r_{j0}}\cdot\frac{4}{\pi}\int_0^{\pi/2} Ki_3(\Sigma_j\lambda_{ji})\cos\beta'\, d\beta' \tag{10}$$

Eqs. (5) and (7) become

$$C_{ji} = \frac{r_{j0}}{r_{ji}} B(\Sigma_j r_{j0}, r_{ji}/r_{j0}) \tag{11}$$

$$C_{j0} = A(\Sigma_j r_{j0}, r_{ji}/r_{j0}) + B(\Sigma_j r_{j0}, r_{ji}/r_{j0}) \tag{12}$$

In Eq. (10), the transformation $\sin\beta = (r_{ji}/r_{j0})\sin\beta'$ has been used.

When $j = n$, $\lambda_{ni} = \infty$ and $C_{ni} = 0$.

When $j = 1$, there are three possibilities:

(i) If the central region is fuel, Eqs. (11) and (12) apply.

(ii) If the central region is the same moderator as region 1, then $r_{ji}/r_{j0} = 0$ and $C_{10} = A(\Sigma_1 r_{10}, 0)$, having been represented by Thie (*5*).

(iii) If the central region is a void region of radius, r_{1i}, separated from the moderator of region 1 in some manner, then

$$\frac{\lambda_{10}}{r_{10}} = \begin{cases} 2\cos\beta, & \beta_t \leqq \beta \leqq \pi/2 \\ 2\left[\cos\beta - \sqrt{\left(\dfrac{r_{1i}}{r_{10}}\right)^2 - \sin^2\beta}\right], & 0 \leqq \beta \leqq \beta_t \end{cases} \tag{13}$$