Letters to the Editors

Manganese Bath Measurements of η of Pu^{239}

In 1959 thermal values of η, the yield of fission neutrons per absorption, were measured by the manganese bath technique. This experiment, and the results for U^{233} and U^{235} have been reported in detail (*1*, *2*). Samples of Pu^{239} were also included in the series of measurements but only preliminary results for this isotope have been quoted (*3*, *4*). The difficulty in obtaining a value at 2200 meters/sec from the measurement has been the appreciable variation of the Pu^{239} η with energy, below the Cd cutoff. While this variation is fairly well measured, to apply the corresponding correction the neutron spectrum used must be well known.

In the previous report (*1*) the spectrum was conventionally assumed to consist of a Maxwellian and a $1/E$ portion, whose relative intensities were determined by gold activation at the sample position. Allowances for uncertainty in temperature (±10%) and in the details of the $1/E$ cutoff function (5 kt ± 50% amply covered the cases considered), were included in the uncertainty quoted. There is some evidence that the Maxwellian plus $1/E$ representation may not be adequate (*5*) and we have hesitated to rely on it for evaluating the correction to 2200 meters/sec for Pu^{239}.

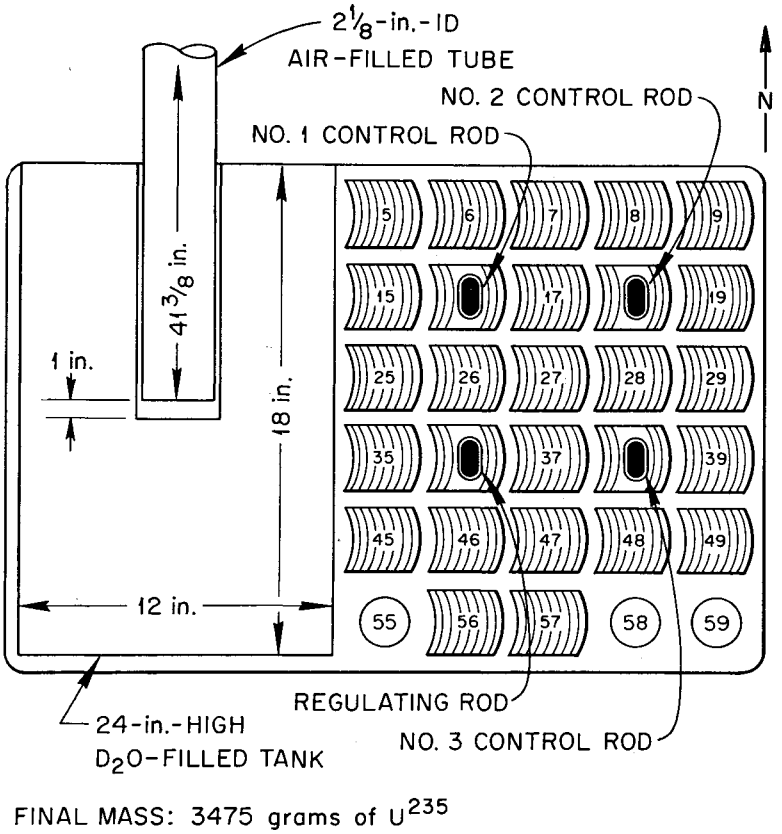

FIG. 1. Reactor loading

An opportunity to measure our beam spectrum directly with a "chopper" velocity selector has failed to present itself. In principle, the spectrum should be calculable (geometry, materials, and reactor loading details are as shown in refs. *1* and *2* and Fig. 1 of this communication), and we would encourage any reader prepared to undertake

such a computation. Meanwhile, interest in the Pu^{239} measurement has continued and we should like to report what data are available.

The technique of measurement has been described previously (*1*, *2*) and the Pu^{239} data is presented in identical notation. The Pu^{239} sample was similar in geometry to samples I and III in ref. *1* as shown in the inset of Fig. 1 of ref. *1*. The tables (I–IV) are analagous to those of ref. *1*. The data used for the variation of the ratio of the total absorption to the fission cross section, $1 + \alpha$, with energy are shown in Fig. 2, analagous to Fig. II.3 of ref. *2*. The assumed uncertainty in $(1 + \alpha_E)/(1 + \alpha_{E^0})$ is represented by the cross hatched area.

The second entry in Table II, our result for a black sample in our thermal spectrum, may be better suited to solid fuelled reactor calculations than the less certain 2200 meters/sec value. The assumption of a spectrum peaked at 0.12 ev as suggested in (*5*) might lower our calculated result for 2200 meters/sec about 1.1%. Accordingly, the lower uncertainty bound in the final result $2.143 \pm \frac{0.010}{0.020}$ has been arbitrarily doubled. The ratio of η for Pu^{239} to the value for U^{235} as determined from our experiment is a bit more precise as the last five corrections of Table I apply to both. Using the doubled lower bound for $\eta(Pu^{239})$ as above, the ratio result for 2200 meters/sec is $\eta(Pu^{239})/\eta(U^{235}) = 1.032$

TABLE I

LIST OF THE CORRECTIONS APPLIED TO THE MEASURED η^* TO OBTAIN η AT A REFERENCE ENERGY CORRESPONDING TO 2200 meters/sec.

Effect	Numerical value of $\eta^*/\eta_{2200\ meters/sec}$
	Sample IV (Pu^{239})
1. Fast multiplication	1.0245 ± 0.0020
2. Energy dependence and transmission[a]	0.9529 ± 0.0032
3. Indirect multiplication	0.9883 ± 0.0020
4. Impurities (mostly Pu^{240})	$0.9943 \pm \frac{0.0001}{0.0017}$
5. Scattering	0.9995 ± 0.0003
6. Resonance absorption in Mn	1.0052 ± 0.0005
7. Structural absorption	1.0060 ± 0.0015
8. Fast leakage	0.9973 ± 0.0015
9. Duct streaming	0.9997 ± 0.0001
10. High-energy parasitic reactions	0.9942 ± 0.0005
Total correction	0.9611 ± 0.0051

[a] Transmission correction alone computed as 0.992 ± 0.002.

TABLE II

RESULTS OF THE EXPERIMENT

	Sample IV(Pu^{239})
η^*, uncorrected experimental measurement	2.0595 ± 0.0020
$\bar{\eta}$, corrected except for energy dependence	2.058 ± 0.010
η, corrected value at 2200 meters/sec	$2.143 \pm \frac{0.010}{0.020}$

TABLE III

COMPARISON OF RESULT OF THIS EXPERIMENT WITH OTHER VALUES OF η OF Pu^{239}, REFERRED TO A VELOCITY OF 2200 METER/SEC

This experiment	$2.143 \pm \frac{0.010}{0.020}$
BNL-325 (1958)[a]	2.10 ± 0.02
HW-69342 (1961)[b]	2.16 ± 0.05

[a] D. J. HUGHES AND R. B. SCHWARTZ, Neutron cross sections, 2nd ed. BNL-325 (1958). See also R. GWIN AND D. W. MAGNUSON, *Nuclear Sci. and Eng.* **12,** 359–363 (1962).

[b] B. R. LEONARD, JR., Survey of the status of low energy cross sections of fissile nuclides, HW-69342, April 21, 1961. Value derived from $\sigma_{nx} = 1008 \pm 6$b, $\sigma_{n,f} = 754 \pm 9$b, $\nu = 2.89 \pm 0.05$.

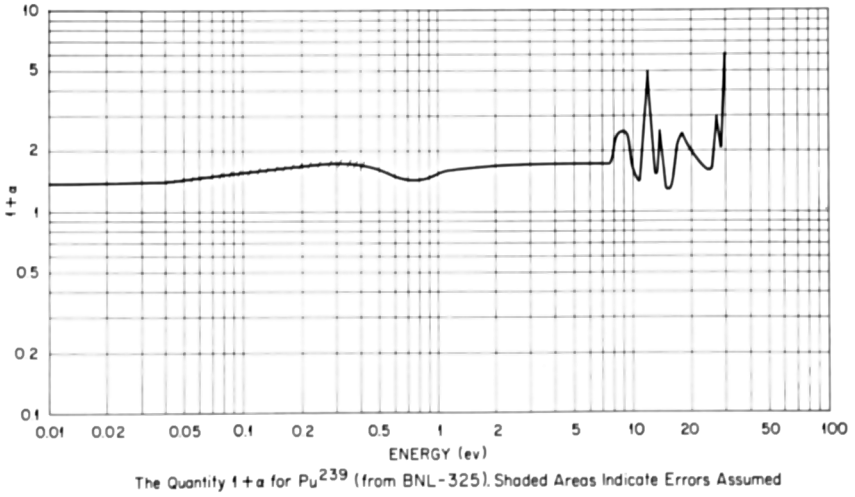

FIG. 2. $1 + \alpha$ vs. E for Pu^{239}

TABLE IV

ANALYTICAL REPORT ON Pu^{239} SAMPLE[a]

Al[b]	9500	Gd < 6	Dy < 24
C	85	B < 0.5	I < 60
O	55	Sm < 60	Hg < 80
Cr	30	Eu < 12	In < 60
Fe	275	Cd < 48	Rh < 60
Ni	20	Li < 2	Er < 24
	Total Pu 98.90 ± .2%		

[a] All values reported in parts per million by weight.

[b] The sample was an alloy containing 1% by weight aluminum.

TABLE V

ISOTOPIC ANALYSIS OF Pu^{239} SAMPLE

Pu^{238}	0.003%
Pu^{239}	97.90%
Pu^{240}	2.01%
Pu^{241}	0.06%

$\pm \frac{0.006}{0.009}$. Similarly, the ratio for Pu^{239} and U^{233} is calculable but redundant.

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Effect of Zero Gradient Boundary Conditions on Cell Calculations in Cylindrical Geometry*

In a recent letter, Thie (*1*) has pointed out the disagreement between computations by various methods of the disadvantage factors of cylindrical rods in certain square lattices. The lattices considered are tightly packed and have low density moderators. The methods which appear to give the most accurate values are a Monte Carlo treatment and Amouyal's method (*2*) which are in approximate agreement. One of the methods with results in disagreement with these values is a P_3 approximation which uses reflecting boundary conditions on a cylindrical surface such that the area of the cylindrical cell is identical with that of the square cell of the physical lattice. An analogous P_1 approximation may also be carried out for the cylindrical cell. Disadvantage factors by these four methods for four of Thie's lattices are shown in Table I. The P_3 values represent an independent check on Thie's calculation in agreement with his results. The disagreement between the P_3 value for reflecting boundary conditions and the Monte Carlo value is small for Lattice 1, but substantial for the smaller radius of Lattice 4, and has the anomalous feature that it differs in the opposite sense from that of the P_1 approximation and by a larger amount. This type of discrepancy has been noted by Newmarch (*3*) who has shown that the disadvantage factor for such a P_3 approximation does not approach unity as the cell dimension becomes small. Similar considerations have also been given by Daitch *et al.* (*5*).

TABLE I

DISADVANTAGE FACTORS BY VARIOUS METHODS

Lattice	Monte Carlo	Amouyal	P_1	P_3 (Reflecting boundary conditions)
1	1.135	1.170	1.051	1.166
2		1.169	1.039	1.189
3		1.155	1.036	1.207
4	1.137	1.159	1.030	1.265

Several points should be made about the reflecting boundary conditions applied on the cylindrical surface. As Thie indicates, these conditions are an artificiality which cannot be realized physically in the case of neutrons. The physical surfaces for which reflecting conditions are implied by the geometry are the flat surfaces of the square cell.

In contrast to the case of plane geometry, reflecting boundary conditions on the cylindrical surface do not imply that the derivative of the scalar flux vanishes, except in the special case of the P_1 approximation. For the lattice in question reflecting conditions imply a large and unrealistic value of this derivative. Figure 1 shows a flux plot across the cylindrical cell for Lattice 4. As the homogeneous case is approached the physical flux must become flat across the cell, and it is clear that a flux shape like that of Fig. 1 cannot be applicable.

In the notation of (*4*), the reflecting boundary conditions for a P_3 approximation have the form

$$\psi_{1,1} = \psi_{3,1} = \psi_{3,3} = 0. \tag{1}$$

The component $\psi_{1,1}$ is the neutron current and the condition $\psi_{1,1} = 0$ must be retained in any cell calculation. The second condition suggested by the above considerations is $d\psi_{0,0}/dr = 0$, $\psi_{0,0}$ being the scalar flux.[1] A third condition may be obtained in several ways; the results have proved insensitive

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] Cell boundary conditions using these two conditions have been used previously (compare ref. *5*).