NUCLEAR SCIENCE AND ENGINEERING: 13, (1962)

Letters to the Editors

Temperature Histories of Internally Heated Thin Bodies Cooled by Convection and/or Thermal Radiation

The heat generated in bodies situated near nuclear reactors due to gamma radiation is often dissipated simultaneously by convection and thermal radiation heat transfer. This applies to many components of nuclear devices operating in the atmosphere or space, such as nuclear ramjets, rockets, and power systems. Presented here are analytic solutions for temperature transients in "thin body" components due to the combined action of known heat generation and cooling (or heating) by convection and/or thermal radiation. A "thin body" has no temperature or heat source strength variations throughout its volume. In practice, a body of high thermal conductivity and small thickness along any heat flow direction may often be considered a "thin body," e.g., thin-walled plates, tubes and shells cooled at the interior and/or exterior surfaces.

The rate of temperature rise with time, dT/dt, in a thin body of volume V, density ρ, and specific heat c is a function of (1) the heat generation rate per unit volume Q, (2) the internal and external convective cooling over areas A_i and A_e with heat conductances h_i and h_e to sink temperatures T_i and T_e (e.g., a tube with an internal cooling fluid subject also to external aerodynamic cooling), and (3) the thermal radiation cooling over area A_r (with view factor F, emissivity ϵ, and Stefan-Boltzmann constant σ) to sink temperature T_r (e.g., to space), as shown in Eq. (1) below (in heat flow units, such as Btu/hr):

Rise in body heat content	=	Heat generation	−	Internal convection cooling	−	External convection cooling	−	Thermal radiation cooling
$V\rho c(dT/dt)$	=	QV	−	$h_i A_i(T - T_i)$	−	$h_e A_e(T - T_e)$	−	$FA_r\sigma\epsilon(T^4 - T_r^4)$

(1)

Combining the inputs into three positive terms X, Y, and Z, Eq. (1) becomes:

$$\frac{dT}{dt} = X - YT - ZT^4 \quad (2)$$

where

$$X = \frac{Q}{\rho c} + \frac{h_i A_i T_i + h_e A_e T_e + FA_r\sigma\epsilon T_r^4}{V\rho c}$$

$$Y = \frac{h_i A_i + h_e A_e}{V\rho c}$$

$$Z = \frac{FA_r\sigma\epsilon}{V\rho c}$$

Even though, in general, X, Y, and Z are functions of time and temperature, a maximum body temperature T_{max} may exist. This is found without integration by putting $dT/dt = 0$ in Eq. (2), i.e.,

$$X - YT_{max} - ZT_{max}^4 = 0 \quad (3)$$

T_{max} is the real positive root of Eq. (3), but its existence depends on the time-varying functions in the inputs. For instance, if Q increases with time at a faster rate than the other inputs, the temperature rises continually, and no maximum is reached.

Solutions to Eqs. (2) and (3) are given below for several cases in which Eq. (2) is separable. Throughout, T_0 is the thin body temperature at time zero.

1. Combined Convection and Thermal Radiation Cooling: Constant X, Y, Z

$$Zt = \frac{1}{4a^3 + U}\ln\frac{T_0 - a}{T - a} + \frac{1}{4b^3 + U}\ln\frac{T_0 - b}{T - b} + \frac{C'}{2}\ln\frac{T_0^2 - 2T_0\sqrt{l} + k + l}{T^2 - 2T\sqrt{l} + k + l}$$
$$+ \frac{C'\sqrt{l} - D'}{\sqrt{k}}\left[\tan^{-1}\left(\frac{T_0 - \sqrt{l}}{\sqrt{k}}\right) - \tan^{-1}\left(\frac{T - \sqrt{l}}{\sqrt{k}}\right)\right]$$

$$T_{max}\ (\text{equilibrium}) = a$$

where

$$a = -\sqrt{l} + \sqrt{-l + 2\sqrt{l^2 + (W/4)}}$$
$$b = -\sqrt{l} - \sqrt{-l + 2\sqrt{l^2 + (W/4)}}$$
$$C' = -\left(\frac{1}{4a^3 + U} + \frac{1}{4b^3 + U}\right)$$
$$D' = \frac{-1}{ab} - \frac{(k + l)}{ab}\left(\frac{b}{4a^3 + U} + \frac{a}{4b^3 + U}\right)$$
$$k = 3(l_1 - l_2)^2/[-l + 2\sqrt{l^2 + (W/4)}]$$
$$l_1 = \frac{U^{2/3}}{4\sqrt[3]{2}}\sqrt[3]{1 + \sqrt{1 + \frac{256W^3}{27U^4}}}$$
$$l_2 = \frac{U^{2/3}}{4\sqrt[3]{2}}\sqrt[3]{1 - \sqrt{1 + \frac{256W^3}{27U^4}}}$$
$$l = l_1 + l_2$$
$$U = Y/Z$$
$$W = X/Z$$

2. Thermal Radiation Cooling Only ($Y = 0$)*: Constant* X, Z

$$2pXt = \psi(pT) - \psi(pT_0)$$

$$T_{\max} \text{ (equilibrium)} = 1/p$$

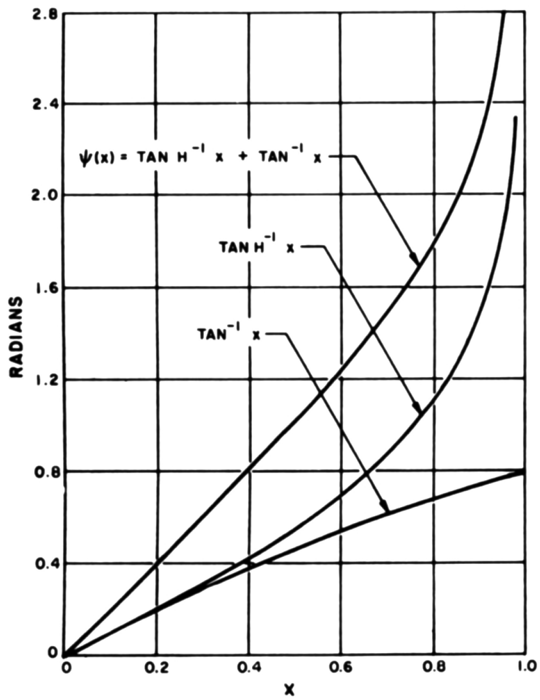

FIG. 1. Functions for thin body cooling by thermal radiation only.

where

$$p = (Z/X)^{1/4}$$

$$\psi(pT) = \tanh^{-1}(pT) + \tan^{-1}(pT)$$

Figure 1 shows curves of the ψ, $\tanh^{-1}$, and $\tan^{-1}$ functions from which the time–temperature relation is obtained directly.

3. Convection Cooling Only ($Z = 0$)*: Time-Varying* X, Y

$$T = \frac{T_0 I_0}{I} + \frac{1}{I}\int_0^t XI\, dt$$

$$T_{\max} \text{ (if existing)} = X/Y$$

where

$$I = \exp \int Y\, dt$$

$$I_0 = \text{value of } I \text{ at } t = 0$$

4. Convection Cooling Only ($Z = 0$)*: Time-Varying Heating* ($Q/\rho c$) *with Constant* Y, S

$$T = T_0 e^{-Yt} + \frac{S}{Y}(1 - e^{-Yt}) + e^{-Yt}\int_0^t Q(t) e^{Yt}\, dt$$

$$T_{\max} \text{ (if existing)} = (Q(t) + S)/Y$$

where

$$Q(t) = \text{time-varying } (Q/\rho c)$$

$$S = (h_i A_i T_i + h_e A_e T_e)/V\rho c$$

a. $Q(t) = Q/\rho c = constant$

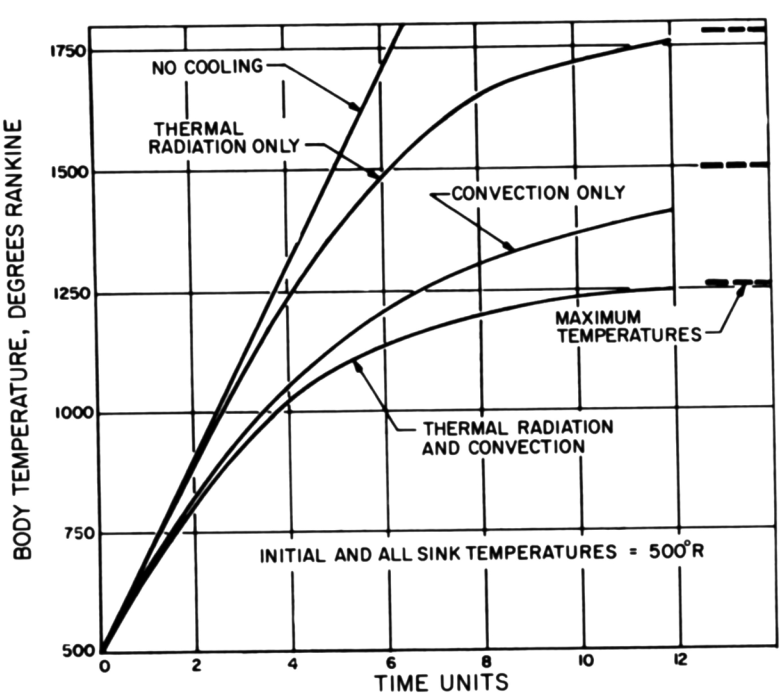

FIG. 2. Thin body heating with various simple-step inputs

$$T = T_0 e^{-Yt} + N_\infty(1 - e^{-Yt})$$

$$T_{\max}\ (\text{equilibrium}) = N_\infty = [(Q/\rho c) + S]/Y$$

b. $Q(t) = Q_\infty(1 - e^{-\alpha t})$

$$T = T_0 e^{-Yt} + N_\infty(1 - e^{-Yt}) - [Q_\infty(e^{-\alpha t} - e^{-Yt})/(Y - \alpha)]$$

$$T_{\max}\ (\text{equilibrium}) = N_\infty = (Q_\infty + S)/Y$$

c. $Q(t) = \beta + \gamma t (\text{no } T_{\max})$

$$T = T_0 e^{-Yt} + \{[1 - e^{-Yt}][Y(S + \beta) - \gamma]/Y^2\} + (\gamma t/Y)$$

5. *No Cooling—Time-Varying Q and Temperature-Varying ρc*

$$\int_{T_0}^{T} \rho c\, dT = \int_0^t Q\, dt$$

The analytical solutions for constant (simple-step) inputs were compared in a numerical example. The following values were selected (in Btu, °R, ft, and arbitrary time units):

$$Q/\rho c = 200$$

$$(h_i A_i T_i + h_e A_e T_e)/V\rho c = 100$$

$$F A_r \sigma \epsilon T_r^4/V\rho c = 1.25$$

$$T_0 = T_i = T_e = T_r = 500$$

Figure 2 shows the resulting heating curves for: combined thermal radiation and convection (1), thermal radiation only (2), convection only (4a), and no cooling (5). Any finite amount of cooling produces an equilibrium temperature, which is not exceeded. This example indicates general trends only; particular environmental factors will affect the relative importance of convective and radiant cooling.

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Received August 14, 1961
Revised January 19, 1962

An Improved Technique for the Determination of Short-Lived Fission-Product Gases*

A number of improvements have been made in the radiochemical technique for determination of short-lived fission gases since publication in the August, 1961, issue of *Nuclear Science and Engineering* (1). In the original procedure, release rates of krypton-89, xenon-137, xenon-140, and xenon-141 were determined during irradiation of fuel specimens by collecting the long-lived decay products of these gases in a trap containing pads of stainless steel mesh. The daughter products (strontium-89, cesium-137, barium-lanthanum-140, and cerium-141) were etched from the pads of mesh and analyzed radiochemically. In the improved procedure, the daughter products are collected on a charged rod and analyzed directly by gamma-ray spectrometry.

The design of the new trap is shown in Fig. 1. It is constructed of stainless steel, and all closures are made by welding except for the removable assembly at the top of the trap. The gas stream enters the trap at the bottom, passes over the charged rod, and exits at the top. The rod is $\frac{1}{8}$-in. stainless steel, and a negative potential of 1000 volts is applied to it during operation. Positively charged daughter products are collected on the rod as the short-lived fission gases decay in passing through the trap. To remove the charged rod for analysis, the cap at the top is unbolted, the electrical connection is broken at the banana plug, and the insulated plug through which the electrical lead passes is unscrewed and lifted out.

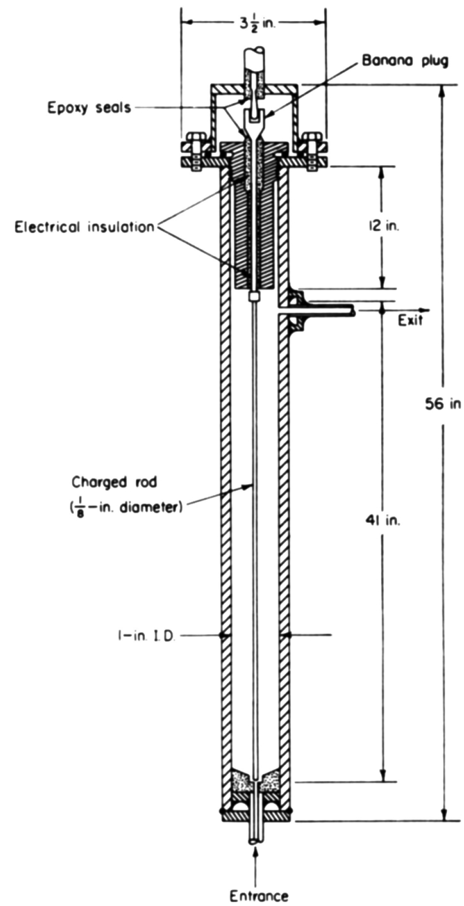

FIG. 1. Charged rod daughter trap

Radiochemical analyses of the solid daughter products are greatly simplified with the improved trapping system. This is due to the possibilities of more rapid removal of the trap from the system and of performing gamma-ray spectrometry directly on portions of the collecting rod. Only 10 to 15 min are required to remove the collecting rod from the trap and to cut it into 1-in. long sections which can be assayed in a gamma-scintillation well-crystal spectrometer. This permits the analyses to be performed on shorter-lived daughter products (such as 14.9-min rubidium-89), thus increasing the number of short-lived gases that can be determined. The costs of analysis are also considerably reduced, since radiochemical separations are not required when gamma-ray pulse height analysis combined with resolution of decay curves can provide the necessary data.

Currently, release rates are being determined for the

* Work performed under AEC Contract W-7045-eng-92.