TABLE I

VARIATION OF FLUX AND DENSITY DEPRESSION WITH FOIL THICKNESS

τ_0	δ_ϕ	δ_ϕ^*	$\delta_\phi/\delta_\phi^*$	δ_ρ	δ_ρ^*	$\delta_\rho/\delta_\rho^*$
0.0001	0.0004498	0.0004544	0.9900	0.0005554	0.0005649	0.9832
0.001	0.003478	0.003523	0.9871	0.004255	0.004350	0.9782
0.01	0.024593	0.025043	0.9820	0.029593	0.030528	0.9694
0.1	0.145359	0.149567	0.9719	0.168832	0.177251	0.9525

depression is serious for foils of quite small radius, more sophisticated calculations are required. On the other hand, the incident flux depression in graphite is an order of magnitude smaller than the self-shielding, for gold foils 0.5 in. in diameter and a few mils thick, so that for graphite, or heavy water, the present type of treatment is probably adequate.

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Ratio of Fundamental and Second Order Harmonics in Flux for Oscillator Experiments with Varying Amplitude of Reactivity Insertion

In designing oscillator rods it is important to know where the limit in terms of reactivity lies if one is to be assured of a linear response. If the reactivity is small enough, the generated flux variation δn will be small in comparison with the steady state flux level n_0, and one is allowed to make the linear assumption in the kinetic equations. In this case, for each frequency generated by the rod, only one flux component will be observed having the same frequency as the reactivity insertions by the rod.

It is easy to conceive of a situation in which the flux variations cannot be treated assuming linearity in the kinetic equations. That is, for a given rod frequency, one will observe several frequencies in the flux. Two cases can be practically visualized where this phenomenon will occur.

First, reactors which exhibit a peaking in the transfer function at power; that is, a region of tendency towards resonance due to the coupling of reactivity and physical motion in the core. In such a region the amplification will be higher and a "weak" oscillator rod might still be strong enough to generate a sizable second harmonic in the flux. Secondly, with the increasing number of reactors being built for power production, it is conceivable that one might not consider it necessary to design a special oscillator rod but would oscillate a control rod instead. This rod might be quite powerful in terms of reactivity and could generate a sizable second harmonic.

In order to analyze oscillator data with harmonic content it is necessary to obtain analytical expressions of fundamental and higher harmonics as a function of the sinusoidal reactivity insertions.[1] In most practical applications it is sufficient to obtain expressions for the fundamental and second harmonic.

In a previous paper by the author (*1*, *2*) expressions have been developed for the generalized case where nonlinear stability considerations were of prime interest.

In this note the pertinent expressions have been extracted to obtain the amplitude ratio $a = \left| X_2/X_1 \right|$ of second order harmonic and fundamental with varying amplitude K_{ex} of reactivity insertions. These equations are applicable to any reactor provided the linear frequency response is known over a range equal to twice the nonlinear range of interest. This derivation is only valid for K_{ex} considerably below prompt critical. Furthermore, the frequencies should not be too high in order to avoid the appearance of spatial modes in the flux (*3*). One nice aspect of this analysis is that all the data necessary to obtain information on nonlinear behavior is linear data.

After some straightforward algebraic manipulations with Eq. (33) (*1*) and solving for the ratio of second order harmonic to fundamental, we obtain for a zero power reactor:

$$\left|\frac{X_2}{X_1}\right| = a = \left| \frac{ZP(2j\omega)K_{ex}/2}{1 + \{ZP(2j\omega)ZP(j\omega) - 2ZP(j\omega)\mathrm{Re}[ZP(j\omega)]\}(K_{ex}/2)^2} \right| \quad (1)$$

where:

X_1 = amplitude of fundamental flux component, half peak to peak
X_2 = amplitude of second order harmonic in flux, half peak to peak

[1] In most practical cases one has an oscillator rod which does not generate a pure sinusoidal reactivity. The motion might be sinusoidal, but the reactivity is not. One will have for each mode in the input a corresponding mode of the same frequency in the flux. What one measures will be a superposition of these modes. By Fourier analysis one can obtain the individual components and one actually gets information on several frequencies with one experiment. In the extreme, one can consider a random disturbance in the reactor as a Fourier integral, and in principle obtain information on an infinite amount of frequencies. But as Dr. Bethe pointed out in an early meeting at Argonne concerned with Reactor Safety, "You put a mess in and you get a mess out." Efforts are therefore made to make the reactivity insertion as purely sinusoidal as possible, because the processing of the flux data from nonsinusoidal oscillator experiments would become unreasonably cumbersome.

$$ZP(j\omega) = \frac{1 - j\omega \sum_i \beta_i/(j\omega + \lambda_i)}{j\omega[l + \sum_i \beta_i/(j\omega + \lambda_i)]}$$

zero power transfer function

β_i = fraction of neutrons that are delayed in ith group
λ_i = decay constant for ith group
K_{ex} = $K_{effective} - 1$ excess reactivity
l = effective prompt neutron lifetime
ω = frequency (rad/sec)
$\mathrm{Re}[ZP(j\omega)]$ = real part of $ZP(j\omega)$

A quantity of interest is the amount of K_{ex} necessary to generate a given ratio of second order harmonic to fundamental $|X_2/X_1|$. Equation (1) represents a quadratic vector equation in $K_{ex}/2$ which leads for a given ratio of "a" to two solutions. Only one solution is applicable.

$$\left(\frac{K_{ex}}{2}\right)^2 = -\frac{b}{2m}\left[\frac{1}{2}\left(\frac{4mc}{b^2}\right) + \frac{1}{8}\left(\frac{4mc}{b^2}\right)^2 + \cdots\right] \qquad (2)$$

where:

$m = a^2(\beta_x^2 + \beta_y^2)$
$b = 2\beta_x a^2 - (\alpha_x^2 + \alpha_y^2)$
$c = a^2$
$\alpha = ZP(j\omega) = \alpha_x + j\alpha_y$
$\beta = ZP(2j\omega)ZP(j\omega) - 2ZP(j\omega)\ \mathrm{Re}\,[ZP(j\omega)] = \beta_x + j\beta_y$

The second solution has to be discarded because it leads to values of K_{ex} near prompt critical for which the assumptions made in the original derivation become invalid.

Similar to Eq. (1), we form the ratio "a" of second order harmonic and the fundamental for a reactor at power.

$$\left|\frac{X_2}{X_1}\right| = a$$

$$= \frac{LP(2j\omega)[1 + n_0PK(j\omega)LP(j\omega)]K_{ex}/2}{1 + \left\{\begin{array}{l} LP(2j\omega)LP(j\omega) + LP(2j\omega)n_0PK(j\omega)[LP(j\omega)]^2 \\ + n_0PK^*(j\omega)LP^*(j\omega)LP(2j\omega) \\ \cdot\{LP(j\omega) + n_0PK(j\omega)[LP(j\omega)]^2\} \\ - 2LP(j\omega)\ \mathrm{Re}[LP(j\omega) + n_0PK(j\omega)\,|LP(j\omega)|^2] \end{array}\right\}(K_{ex}/2)^2} \qquad (3)$$

where:

P = power level in Mw
$K(j\omega)$ = $(\nabla k/k)/Mw$ power coefficient
n_0 = steady state flux (Since we are interested in reactivity changes with respect to the steady state flux n_0, we are free to set $n_0 = 1$.)

$$LP(j\omega) = \frac{ZP(j\omega)}{1 - ZP(j\omega)PK(j\omega)} = \frac{\delta n}{n_0}\Big/ K_{ex} \qquad (4)$$

Equation (4) follows from the following symbolism assuming linearity (*2*).

$$\delta k(j\omega) = K_{ex}(j\omega) + n_0PK(j\omega)\frac{\delta n(j\omega)}{n_0} \qquad (5)$$

The second term in Eq. (5) assumes that changes in reactivity are linear functions of the power variation. We have written both the zero power transfer function $ZP(j\omega)$ and the resulting incremental reactivity in the frequency domain and we can represent $ZP(j\omega)$ and Eq. (5) symbolically in a feedback system shown in Fig. 1 (*5*). In this symbolic notation the feedback is positive. This is done in order to retain the physical meaning of the sign of the power coefficient.

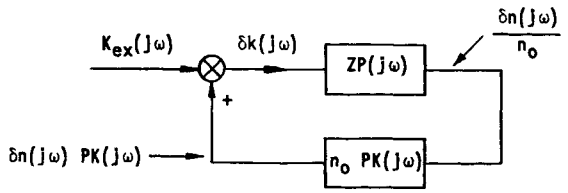

FIG. 1. Feedback system

In an analog manner, as for the zero power case, we can solve for K_{ex} necessary to obtain a given ratio $a = |X_2/X_1|$ for a reactor under power.

For the terms in Eq. (2) we get (and again the second solution has to be discarded):

$m = a^2(\lambda_x^2 + \lambda_y^2)$
$b = 2a^2\lambda_x - (\delta_x^2 + \delta_y^2)$
$c = a^2$
$\delta = LP(2j\omega)[1 + n_0PK(j\omega)LP(j\omega)]$
$\lambda = LP(2j\omega)LP(j\omega) + LP(2j\omega)n_0PK(j\omega)[LP(j\omega)]^2$
$+ n_0PK^*(j\omega)LP^*(j\omega)LP(2j\omega)$
$\{LP(j\omega) + n_0PK(j\omega)[LP(j\omega)]^2\}$
$- 2LP(j\omega)\ \mathrm{Re}\,[LP(j\omega) + n_0PK(j\omega)|\,LP(j\omega)|^2]$

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* indicates conjugate complex function.

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Simplified Calculation on Thermal Transient of a UO_2 Fuel Rod

The thermal behavior of a cylindrical UO_2 fuel rod is characterized by many parameters, namely, a high thermal resistance and a relatively small capacitance of the ceramic