region including gains and losses from xenon, temperature effects and control rod movement in that subvolume) remained constant with time; and by similar control maneuvers assume that the "pseudo k_∞" of subregion B is also held constant with time. Then no flux distortion effects due to the xenon changes taking place within subregions A or B would occur, and the feared oscillation could not develop. Thus, use of the local buckling, or "pseudo k_∞" concept can make the "inherent instability" phenomenon academic if one is only astute enough to move the right amount of rod in the right place at the right time.

Here the concept of "trend" control makes the astute control maneuver simple. A monitor point within subvolume A sensitive to neutron flux level informs the operator whether that region is increasing or decreasing in flux level. With the total reactor power held constant, such a local change in flux corresponds to a very long reactor period measurement thus indicating a local change in reactivity. Therefore, a control tip within subvolume A is inserted or withdrawn to keep the local flux indicator, and thus the local "pseudo k_∞," at a near constant level. Similarly, the trend is noted in region B, and compensating rod moves are made in region B.

Probably the main reasons these concepts work in practice (which they have for many years) are that: (i) they are based on sound reactor physics concepts; and (ii) reaction times of the instruments and operators are very short compared to the 9.2 and 6.7 hr half-lives of Xe^{135} and its precursor I^{135}, respectively.

One other point—the possibility that these slow (order of hours) xenon oscillations could threaten reactor safety (*4*) would imply that the reactor under study was not adequately instrumented and operated to control those flux distortions which frequently arise due to other operating factors. In other words, xenon spatial oscillations should be regarded as an operating efficiency factor, not as a nuclear safety hazard.

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Thermal Neutron Flux in a Cell with Temperature Discontinuities

In solving the reactor equations, it is customary to treat the thermal neutrons as a monoenergetic group, with the slowing down density as a source term. By taking appropriately averaged cross sections for this equivalent thermal group, reasonably accurate calculations can be made for a homogeneous reactor. Many design problems in practice, however, involve adjacent regions with markedly different temperatures. The lattice cell of a reactor may contain high-temperature fuel elements, while the coolant may be below the boiling point of water. The interpretation of lattice measurements, in which the part of the lattice being studied is held at an elevated temperature, is complicated by uncertainties as to the width of the transition region caused by the temperature discontinuity.

Although a straightforward multigroup approach will solve the problem to any required degree of accuracy, many groups are needed to represent the energy distribution adequately, and the thermal calculation may require more effort than the rest of the analysis. As an alternative for design calculations, the following method employs the formalism of few-group theory while retaining the qualitative features of the neutron distribution in space and energy. The basic procedure is to approximate the actual neutron distribution by a superposition of overlapping thermal groups, one in equilibrium with each region of uniform temperature in the system. Neutrons in the non-equilibrium groups will then make transitions into the local equilibrium group at a rate which can be easily calculated for a heavy gas moderator.

Consider a system made up of N regions of different but uniform temperature and composition. Within the nth region, let $\chi_n(E)$ denote the equilibrium thermal neutron distribution (that is, the steady-state distribution that would exist in an infinite medium) normalized to unity with some convenient cutoff energy. The total flux, as a function of position and energy, will be approximated by

$$\phi(r, E) \cong \phi_1(r)\chi_1(E) + \cdots + \phi_N(r)\chi_N(E) \qquad (1)$$

where $\phi_n(r)$ is the total flux of neutrons at r in the group in equilibrium with region n. The individual group fluxes are defined throughout the system, and in general will have considerable overlap in energy.

Within region 1, the group fluxes will satisfy balance equations of the form:

$$\begin{aligned} &-D_1\nabla^2\phi_1 + \Sigma_{a1}\phi_1 = \Sigma_{12}\phi_2 + \cdots + \Sigma_{1N}\phi_N + q \\ &-D_n\nabla^2\phi_n + (\Sigma_{an} + \Sigma_{1n})\phi_n = 0, \qquad n = 2, \cdots, N \end{aligned} \qquad (2)$$

where D_n and Σ_{an} are the diffusion constant and absorption cross section for neutrons of the nth group, $\Sigma_{1n}\phi_n$ is the rate at which neutrons are transferred from group n to the equilibrium group 1, and q is the slowing down density of fast neutrons, all evaluated in region 1. Corresponding equations hold for each of the other regions. Although diffusion theory has been assumed, transport approximations such as spherical harmonics or S_N theory could also be used.

The transfer cross sections, Σ_{1n}, can be evaluated immediately under the assumption of a heavy gas moderator. To first order in $\mu = 1/A$, the ratio of neutron to moderator mass, the differential scattering cross section can be written (*1*)

$$\begin{aligned} &\Sigma_s(E \to E') \cong \Sigma_0\delta(E - E') \\ &+ \mu\Sigma_0 \cdot (E' + E)\sqrt{(E'/E)}[\delta'(E' - E) + kT\delta''(E' - E)] \end{aligned} \qquad (3)$$

where Σ_0 is the bound atom cross section, T is the Kelvin temperature of the moderator, and k is the Boltzmann

constant. The average energy loss per scattering collision in medium 1 is then

$$\overline{\Delta E_1} = \int_0^\infty dE'(E' - E)\Sigma_s(E \to E') \Big/ \int_0^\infty dE'\Sigma_s(E \to E') \cong 2\mu(E_1 - E) \tag{4}$$

where $E_1 = 2kT_1$ is the equilibrium energy in medium 1; that is, neutrons at E_1 on the average neither gain nor lose energy during their next collision.

For neutrons in group n, which must lose an amount of energy $E_1 - E_n$ on the average in order to reach equilibrium, the mean number of collisions required is

$$(E_1 - E_n)/\overline{\Delta E_1} = 1/2\mu \tag{5}$$

Since group n neutrons undergo scattering collisions at the rate $\Sigma_s\phi_n$, and $1/2\mu$ collisions are required to transfer a neutron to group 1, the rate at which neutrons are transferred, $\Sigma_{1n}\phi_n$, is $\Sigma_s\phi_n/(1/2\mu)$. The transfer cross section is therefore

$$\Sigma_{1n} = 2\mu\Sigma_s \tag{6}$$

which can also be written $\xi\Sigma_s$, where ξ is the usual logarithmic energy loss at high energies. If the moderator contains more than one isotope, the transfer cross sections for each component will combine additively. The remaining group constants can be obtained by averaging Σ_a and D over the corresponding group spectra (for a heavy gas model $D(E)$ can be taken as constant if the calculation is only to first order in the mass ratio).

With the group constants determined, the group equations (2) can be solved analytically in one-dimensional geometries. Defining

$$\eta_1^2 = \Sigma_{a1}/D_1, \qquad \eta_n^2 = \Sigma_{1n}/D_1,$$
$$\nu_n^2 = (\Sigma_{an} + \Sigma_{1n})/D_n$$

within region 1, the corresponding group fluxes are given by

$$\phi_1(r) = A_1 I_0(\eta_1 r) + B_1 K_0(\eta_1 r) + \frac{q}{\Sigma_{a1}} + \sum_{n=2}^{N} \frac{\eta_n^2}{\eta_1^2 - \nu_n^2}\phi_n(r)$$
$$\phi_n(r) = A_n I_0(\nu_n r) + B_n K_0(\nu_n r), \qquad n = 2, \cdots, N \tag{7}$$

where I_0 and K_0 are, for example, modified Bessel functions for the case of cylindrical geometry. The $2N^2$ constants A_n, B_n are determined by requiring continuity of flux and current for each group at each internal boundary and the appropriate albedo at the center and outermost boundary.

For a cell consisting of a fuel rod surrounded by moderator, neutrons entering the fuel would require on the order of 100 collisions to reach equilibrium, and would either be absorbed or escape from the fuel long before this occurred. However, selective absorption will distort the exit spectrum; calculations by Stuart (2) indicate that it is accurately represented by a shifted Maxwellian distribution corresponding to a higher neutron temperature. The preceding analysis can therefore be applied to find the distribution of neutrons between the equilibrium and exit spectra as a function of position in the moderator, as well as the entrance spectrum, by using an appropriate albedo matrix to describe the fuel rod.

In order to evaluate the appropriateness of the group model, one can apply it to the case of a uniform medium containing a temperature discontinuity, for which an analytical solution is available (3). Assuming isotropic scattering, no sources or absorption, and taking the temperature discontinuity at $x = 0$ in a plane perpendicular to the x axis, the group fluxes are found to be

$$\phi_1(x) = \begin{cases} 1 - \frac{1}{2}e^{x/L}, & x < 0 \\ \frac{1}{2}e^{-x/L}, & x > 0 \end{cases}$$
$$\phi_2(x) = \begin{cases} \frac{1}{2}e^{x/L}, & x < 0 \\ 1 - \frac{1}{2}e^{-x/L}, & x > 0 \end{cases} \tag{8}$$

The relaxation length, $L = \lambda_s/\sqrt{6\mu}$, agrees with the result previously obtained by Kottwitz. For this particular case, therefore, the group model reproduces the essential features of the exact solution. If scattering is linearly anisotropic and absorption is present, the relaxation length generalizes to

$$L = \sqrt{D/(\xi\Sigma_s + \Sigma_a)}$$

with

$$D = 1/3(\Sigma_{tr} + \Sigma_a) \tag{9}$$

Analytical solutions for more than a few groups and regions rapidly become unwieldy. However, most few-group computer programs, which numerically integrate the conventional group equations, can be modified to deal with (2) by including up-scattering. In the computation of the flux for a given group, the usual source term will consist of the actual fission source along with neutrons scattered down from higher groups. The necessary modification to such a code simply requires the addition of a term corresponding to neutrons scattered upwards in energy from the lower groups, calculated from the last previous flux iterates. Since these are available within the fast memory for most few-group programs, the necessary recoding is quite trivial and involves no change in the over-all program logic.

There are various alternatives to the intuitive derivation of the group equations given here. For the case of a two-temperature system containing no sources or absorption, Kottwitz has obtained the group equations by requiring that the zeroth and first energy moments of the balance equation be preserved (4). A more general derivation can be given by writing a variational principle for the balance equation and taking expression (1) as a trial function. Using the heavy gas model, Maxwellian fluxes, and adjoint fluxes which are linear in energy, equations of the form (2) can be derived (5) for a two-region system. The coefficients, however, turn out to involve correction terms unless the absorption vanishes, since otherwise the trial spectra chosen do not describe equilibrium neutron groups. A still more general procedure is to write a variational principle without particularizing the energy transfer kernel and to use numerically calculated energy distributions for the direct and adjoint fluxes. This approach has been carried out by Calame and Federighi (6, 7) to provide a general numerical space-energy code in both the diffusion and double P_1 approximations. In this case it is found that the complete space-energy distribution can often be approximated by superimposing only two characteristic spectra, although the physical interpretation then becomes somewhat less direct.

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Local Flux Distributions in ORR Fuel Elements

A detailed study of the thermal neutron flux distribution in the ORR is being made by the Operations Division of the Oak Ridge National Laboratory. Results to date have indicated that rather large flux gradients exist radially across the MTR type fuel element employed in this reactor and that these gradients are strongly dependent on the fuel concentration in the element being investigated as well as that of the adjacent elements.

The technique used to measure flux is the one considered as standard at the ORR (*1*). Basically the method consists of measuring the induced activity in a 20-mil cobalt wire. The wires are put into aluminum holders which are placed in the coolant channels between fuel plates. The wires are then irradiated for one hour at a power level of 20 kw.

Figure 1 shows the geometrical pattern of the standard ORR core, and Table I presents the complete set of fuel weights loaded in the core for each experiment represented by the subsequent figures. Because of the operating schedule of the ORR and the time lapse between data gathering and analysis, loading of identical cores was impossible. A flux traverse along the center plane of the "4" column at a distance 16 in. from the top of the fuel plates is shown in Fig. 2. The traverse is shown at 16 in. because the peak axial flux occurs at approximately 16 in. from the top of the fuel plates. Fluxes have been normalized to the highest measured value in the "4" column at the 16 in. level.

It must be pointed out that 131 g of U^{235} in the shim rod gives the same fuel density in its fuel plates as that for a 200-g fuel element. However, due to the structure of the shim rod, the metal to water ratio of the fuel follower is not the same as it is for a fuel element.

It is interesting to note that the ratio of maximum extrapolated flux to minimum measured flux along the center plane is 2.3 for A-4 and 1.3 for E-4. It is perhaps more interesting to observe that the ratio of maximum extrapolated flux to center flux is 1.4 for A-4, 1.3 for C-4, and 1.1 for E-4. For routine core flux measurement in the ORR, it is common practice to measure fluxes in the geometric center of each fuel element. However, one sees that such a measurement gives a value far below the maximum flux in certain cases. The implications of such a situation

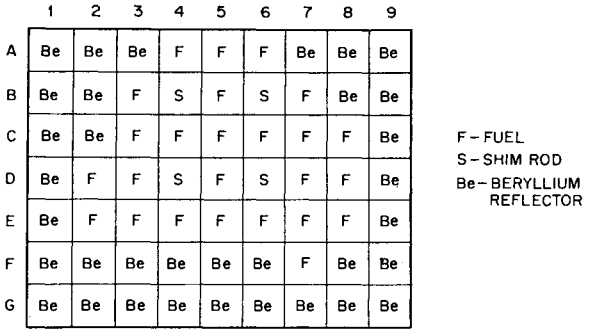

Fig. 1. Core loading pattern of ORR

TABLE I

Fuel Weight in Grams Loaded in ORR for the Various Flux Measurements

Position	Figs. 2 and 3	Fig. 4	Fig. 5
A-4	163	176	169
A-5	169	161	169
A-6	166	168	167
B-3	193	192	192
B-4	142	46	132
B-5	153	140	156
B-6	110	120	102
B-7	189	189	188
C-3	131	112	148
C-4	148	151	147
C-5	163	140	163
C-6	146	200	180
C-7	157	147	157
C-8	161	157	200
D-2	159	179	159
D-3	189	173	175
D-4	91	102	82
D-5	161	138	160
D-6	69	131	60
D-7	188	157	157
D-8	142	170	121
E-2	160	183	157
E-3	189	189	172
E-4	188	188	200
E-5	158	152	179
E-6	159	173	175
E-7	187	156	158
E-8	162	142	200
F-7	148	158	148