LETTERS TO THE EDITORS

A One-Group Model for Thermal Activation Calculations

A common procedure in performing diffusion theory calculations is to use thermal group cross sections that have been averaged over an asymptotic flux spectrum for the region of interest. If the Wigner-Wilkins spectrum as calculated with the SOFOCATE (*1*) Code is used, the one-group diffusion equation is written as

$$-D_{\mathrm{SOF}}\nabla^2\phi + \Sigma_{\mathrm{SOF}}\phi = S \tag{1}$$

where S is a thermal source term. (The flux gradient spectrum used by SOFOCATE for averaging the diffusion coefficient has the same energy dependence as the flux.) The use of regionwise thermal constants with flux and current continuity results in a calculated discontinuity of activation at boundaries between dissimilar media since the microscopic cross section is assumed to have an abrupt change in value at the boundary. In addition, since the regionwise constants do not account for a softening of spectrum approaching a water gap, the peaking with this model may be underestimated by as much as 20%.

Insofar as the quantity of interest in both experimental analysis and nuclear design is the activation of an absorber, it has been suggested by M. Goldsmith and S. Stein of Bettis Laboratory that activation continuity ($\sigma\ \phi$) be used as a boundary condition rather than flux continuity. For a $1/v$ absorber, this may be obtained by using neutron number density rather than flux as a variable since

$$\int \sigma(E)\phi(E)\,dE \propto \int n(E)\,dE = n \tag{2}$$

where $n(E)$ is the energy-dependent neutron density. The absorption term may then be written as

$$\int \Sigma_a(E)\phi(E)\,dE = \overline{\Sigma v}\,n \tag{3}$$

and, if a SOFOCATE spectrum is assumed,

$$\overline{\Sigma v} = \frac{\int \Sigma_a(E)\phi(E)\,dE}{n} = \frac{\int \Sigma_a(E)\phi(E)\,dE}{\int 1/v\phi(E)\,dE} = \frac{\Sigma_{\mathrm{SOF}}}{\overline{(1/v)}_{\mathrm{SOF}}} \tag{4}$$

If the gradient spectrum is also assumed to have the SOFOCATE energy dependence, the thermal group representation would become

$$-\frac{D_{\mathrm{SOF}}}{\overline{(1/v)}_{\mathrm{SOF}}}\nabla^2 n + \frac{\Sigma_{\mathrm{SOF}}}{\overline{(1/v)}_{\mathrm{SOF}}}\,n = S \tag{5}$$

with continuity of neutron density and current. While the discontinuity of the thermal activation is eliminated by this approach, the calculated activation shapes within a region are essentially those of (1) since the characteristic diffusion length is the same in both cases.

Calculations with the SLOP-1 (*2*) multigroup thermal code with upscattering have shown that, while the cross section as a function of position approaches the SOFOCATE value asymptotically (Fig. 1), the diffusion coefficient is much closer to the maxwellian average (Fig. 2). In contrast to the SOFOCATE calculation, SLOP-1 does not assume an identical energy spectrum for the flux and the flux gradient; the calculation of the gradient spectrum is a natural consequence of the multigroup solution. The code edits the diffusion coefficient as

$$D(x) = \int J(E, x)\,dE \Big/ \int \nabla\phi(E, x)\,dE \tag{6}$$

The discontinuities in the diffusion coefficient in Fig. 2 occur where both the current and the flux gradient approach zero and the diffusion coefficient is undefined.

The observation that the SLOP-1 averaged diffusion coefficients are closer to maxwellian in value implies that while the asymptotic flux has a SOFOCATE spectrum the gradient spectrum seems to be much more maxwellian. Plots of the flux and the gradient spectra at various locations in problems containing water channels have shown this to be true. The gradient and the flux spectra are different in shape because the major part of the current in the fuel region is from neutrons originating in the water channel where the spectrum is softer. In addition, Gelbard and Pearson (*3*) have shown that these neutrons diffusing from the water channel into the fuel region lose the higher energy contribution to the spectrum resulting in an even softer spectrum. While the flux in the fuel region is continuously supplemented by fast neutrons slowing down in the fuel region, this source term is relatively flat spatially and does not contribute to the current term.

Using the observations from the SLOP-1 problems to modify (5) results in the diffusion equation of the form

$$-\frac{D_{\mathrm{max}}}{\overline{(1/v)}_{\mathrm{max}}}\nabla^2 n + \frac{\Sigma_{\mathrm{SOF}}}{\overline{(1/v)}_{\mathrm{SOF}}}\,n = S \tag{7}$$

where the coefficients for the leakage term are averaged over a maxwellian spectrum and the coefficients for the absorption term are averaged over a SOFOCATE spectrum. The thermal group representation given in Eq. (7) has been quite successful in duplicating the activation shapes as calculated by SLOP-1. An example of this is given in Fig. 3. Since $\Sigma_{\mathrm{SOF}}/\overline{(1/v)}_{\mathrm{SOF}}$ is similar to $\Sigma_{\mathrm{max}}/\overline{(1/v)}_{\mathrm{max}}$ for most regions of interest, the diffusion length in this mixed number density system is essentially the same as the maxwellian diffusion length. (In fact, if Σ has truly a $1/v$ dependence, this model degenerates to the conventional maxwellian calculation.) This achieves the desired result inferred by

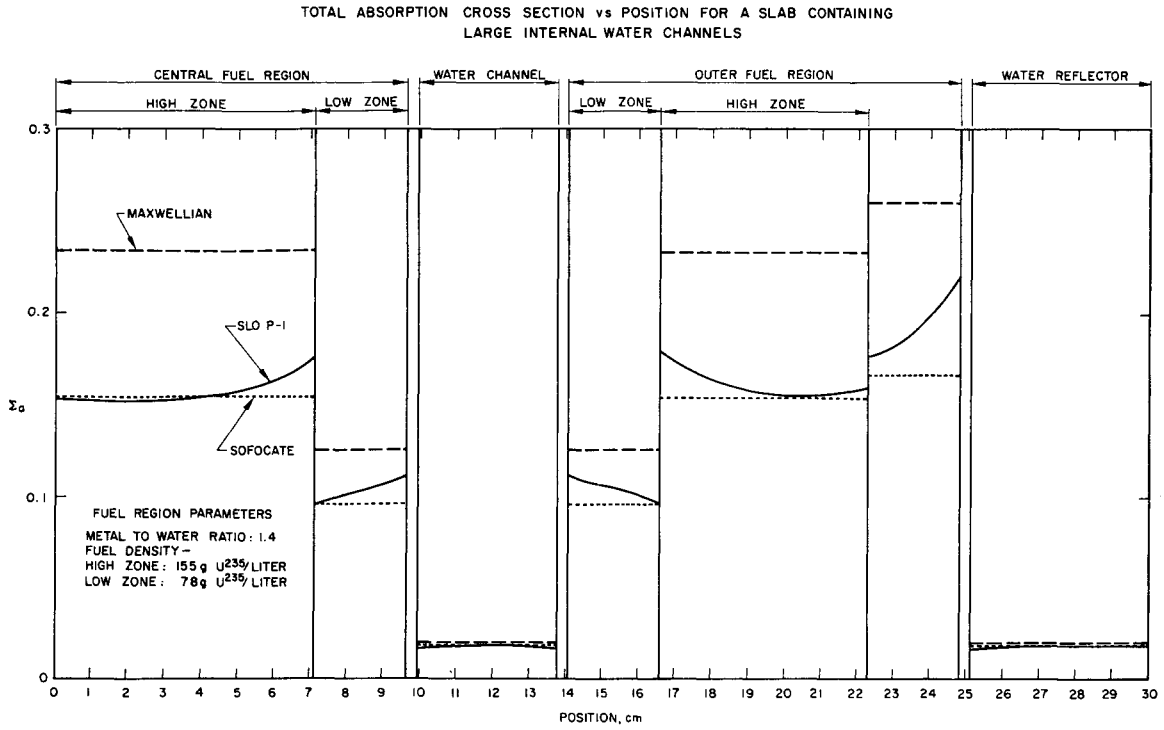

FIG. 1. Total absorption cross section versus position for a slab containing large internal water channels. Comparison of averages over SOFOCATE, maxwellian, and SLOP-1 spectra.

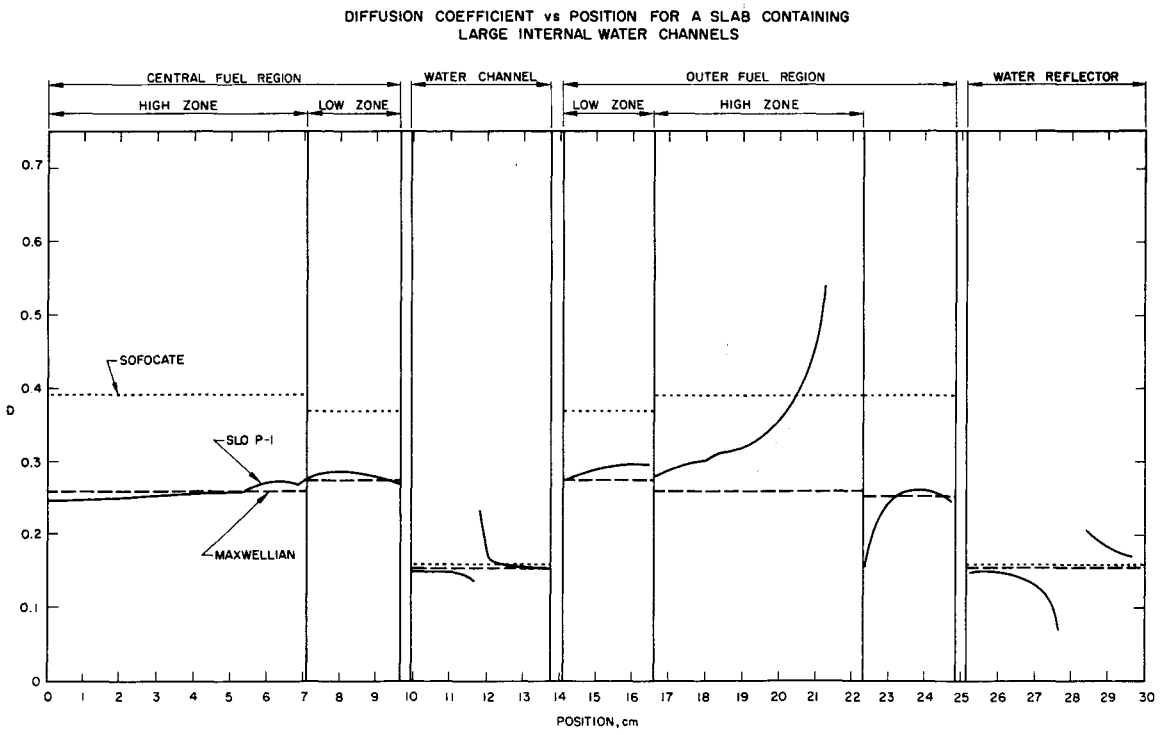

FIG. 2. Diffusion coefficient versus position for a slab containing large internal water channels. Comparison of averages over SOFOCATE, maxwellian, and SLOP-1 spectra.

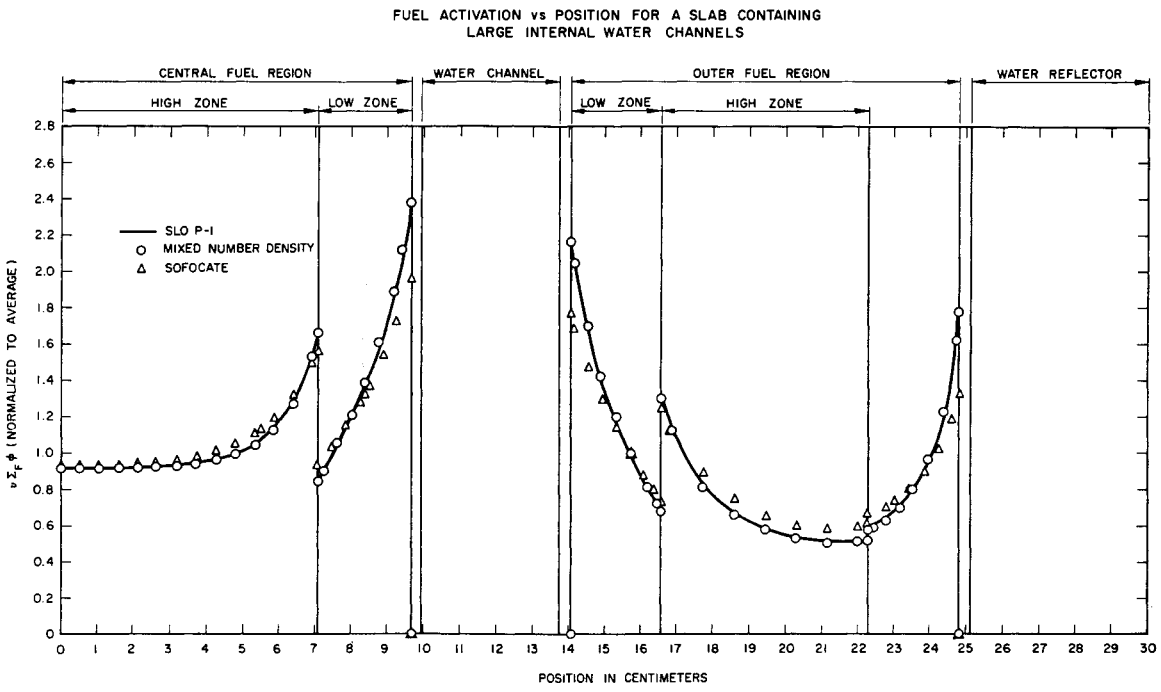

FIG. 3. Fuel activation versus position for a slab containing large internal water channels.

Wright and Feiner (*4*) from the analysis of water-gap peaking experiments. In effect, this representation combines the local SOFOCATE thermal utilization with a maxwellian activation shape.

An additional utility of this model lies in its ready application to existing few-group diffusion theory codes in one, two, or three dimensions. Comparisons of this model to both one and two-dimensional activation experiments are in progress and appear to verify its applicability when the Radkowsky prescription (*5*) is used for evaluating the scattering kernel.

REFERENCES

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4. W. B. WRIGHT AND FEINER, Note on position-dependent spectra. *Nuclear Sci. and Eng.* **6**, 81 (1959).
5. A. RADKOWSKY, Argonne National Laboratory Report, ANL-4476, p. 89 (July, 1950).

R. J. BREEN

Battis Atomic Power Laboratory
Westinghouse Electric Corporation,
Pittsburgh, Pennsylvania
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Let the Reactor Prevent Xenon Instability

The prediction of xenon spatial oscillations in large high-flux reactors has received considerable treatment in the literature by the reactor dynamics approach. It is difficult for most engineers or managers to satisfactorily judge the feasibility of a given reactor design with respect to the complex calculational results. I would like to suggest the use of two other familiar concepts in approaching this problem which are easily understood and which I feel support two practical conclusions:

(1) With proper design and operating flexibility it is practicable to expect good flux distribution control in spite of calculated indications of "instability thresholds" (this statement is in complete agreement, incidentally, with the box summary given in reference *1*, and with the conclusions given in references *2* and *3*).

(2) The timing and detection of the phenomenon are slow relative to human response time, precluding the issue of nuclear safety.

The suggested concepts, whose application is described below, are: (i) local buckling, or what might be called a "pseudo k_∞"; and (ii) very long priods or "trends," indicative in cases of constant total reactor power of increasing or decreasing local bucklings. Use of these concepts is aimed at obviating oscillations rather than describing them mathematically.

Suppose that in some subvolume of the pile, say region A, the net amount of xenon is increasing, whereas it is simultaneously decreasing in a symmetrical subvolume B. One could conclude, "xenon oscillations are starting, so start watching for the flux oscillations." But suppose that by means of astute control rod movements, the "pseudo k_∞" of subvolume A (the net multiplication within that