LETTER TO THE EDITOR

## **On the Fast Effect in Beryllium**

Calculations have been made of the effect of the  $(n, 2n)$  and  $(n, \alpha)$  reactions in a beryllium moderated reactor. Our results show that the  $(n, 2n)$  effect can produce a significant improvement in the neutron economy, but uncertainties in the available experimental data preclude the determination of a meaningful quantitative result. This note stresses the need for more definitive data.

Figure 1 illustrates the dilemma. Shown are the available experimental data on the  $(n, 2n)$  reaction from threshold (1.85 Mev) to 14 Mev. The 4-Mev point of Beyster *et al. (1)* and the data above 7 Mev are the results of sphere measurements which yield nonelastic cross sections. We have obtained the  $(n, 2n)$  cross sections by subtracting the  $(n, \alpha)$  cross sections from the measured nonelastic results using the  $(n, \alpha)$  work of Campbell and Stelson  $(2)$ , Battat and Ribe *(3),* and Sattar *et al.,* (4), and performing a smooth interpolation between their experimental results. Our difficulty involves the interpretation of the experimental results below 4 Mev.

In this region the data consist primarily of the work of Fischer *(5)* and of Marion *et al.* (*6*), the single exception being the one point of Fowler *et al. (7)* at 3.7 Mev. Unfortunately it is difficult to interpret the data in a consistent manner. Fischer concludes from his work that the threshold for the  $(n, 2n)$  reaction is at 2.7 Mev (i.e., that the reaction takes place only through the 2.43 Mev level) and that the cross section increases very rapidly from this threshold. However, there are large quoted experimental errors in his data and his conclusion on the threshold at 2.7 Mev is directly contradicted by Marion *et al. (6, 8)* who, with their time of flight technique, observed neutrons which were too energetic for an  $(n, 2n)$  threshold at 2.7 Mev. Thus, it is difficult to use Fischer's data in a definitive way.

On the other hand, the work of Marion *et al.* also involves a question of interpretation. Their  $(n, 2n)$ cross sections were obtained (by them) by subtracting their measured elastic cross section from their measured total cross section. Now their total

cross-section measurement is completely independent of their elastic cross-section measurement. Different Be targets were used and different methods of normalizing the results were employed. Thus there seems to be no gain in internal consistency resulting from using their total cross sections with their elastic cross sections as against using the total cross sections of others.

Unfortunately Marion *et al.* are in disagreement with the total cross section work of Fowler and Cohn *(9, 10)* and Bockelman *et al. (10, 11).* Furthermore the latter two groups of experimenters have a very large number of consistent experimental points obtained with several different beryllium targets.<sup>1</sup> Thus there appears no reason to favor the total cross sections of Marion *et al.,* and by weight of numbers one might be inclined toward the work of the other experimenters *(9, 10, 11).* Shown in Fig. 1 are the  $(n, 2n)$  cross sections which result when the elastic cross sections of Marion *et al.* are used with the total cross sections of Bockelman *et al.* and Fowler and Cohn. Finally when the total cross sections of BNL-325 are used beyond the range of the data of references  $9$ ,  $10$ , and  $11$  the large  $(n, 2n)$ cross sections obtained appear inconsistent with the sphere data above 4 Mev. This is also shown in Fig. 1.

From the discussion above one may appreciate our dilemma in choosing cross sections to calculate the effect of the  $(n, 2n)$  reaction in a Be reactor. However, in order to obtain estimates of its magnitude and to assess the sensitivity to variations in cross sections we have made calculations based on a number of assumed shapes for the  $(n, 2n)$  cross section as a function of energy. The shapes chosen range from "wildly optimistic" to "very pessimistic" as shown by the curves labeled A, B, and C in Fig.

<sup>&</sup>lt;sup>1</sup> The results of Bockelman *et al.* are in excellent agreement with those of Fowler and Cohn when inscattering corrections based on the measured elastic cross-section angular distributions are made. These corrections have been made by Fowler *(10).* 



FIG. 1. Available experimental data pertaining to the beryllium *(n, 2n)* reaction.

1. In all cases we have assumed the correctness of the 500-600 millibarn results of the sphere measurements between 4 and 11 Mev (although the spread of results at 14 Mev indicates that even these measurements involve difficulties). The important consequence of this assumption is its effect on our choice of curves below 4 Mev rather than its direct effect on the *(n,* 2*n)* contribution above 4 Mev which at any rate is small.

The calculations were performed using a method similar to that of Aline and Harker *(12).* The region above the  $(n, \alpha)$  threshold of 0.71 Mev was broken into 18 neutron groups with the region above 6 Mev taken as the first group. To obtain averages within groups it was assumed that the fission spectrum held within the group. Except for the first group, the importance of which is small, the group widths are quite narrow so that the results are not very sensitive to the choice of a reasonable neutron spectrum *within the groups.* It was assumed that all neutrons emerging from the *(n,* 2*n)* reaction are below the  $(n, \alpha)$  threshold. (This assumption will produce a slight underestimate of both the  $(n, \alpha)$  and *(n,* 2*n)* effects.) Thus, transfer between groups occurs only due to elastic collisions. We calculated  $\sigma_{i \to j}$ , the elastic transfer cross sections, from the expression

$$
\frac{\int_{E_i}^{E_i+\Delta E_i} \phi_i(E) \left[ \int_{\theta(E_j)}^{\theta(E_j+\Delta E_j)} \frac{d\sigma(E,\theta)}{d\Omega} 2\pi \sin\theta d\theta \right] dE}{\int_{E_i}^{E_i+\Delta E_i} \phi_i(E) dE}
$$

where  $E_i$  is the lower limit of the energy group from which the elastic scatter took place,  $\Delta E_i$  is the group width,  $\phi_i(E)$  is the flux in group i,  $d\sigma(E, \theta)/d\Omega$  is the differential scattering cross section,  $\theta(E_i)$  is the maximum angle through which the neutrons can be scattered from group *i* and still have an energy  $E_j$ , and  $\theta$  ( $E_j + \Delta E_j$ ) is the minimum angle through

TABLE I CALCULATED FAST EFFECTS FOR AN INFINITE  $B_0$  Core

DE VORE			
(1) $(n, 2n)$ cross sections		(2) (3) $(n, 2n)$ reactions $(n, \alpha)$ reactions per fission neutron per fission neutron	Be fast effect $1+(2)-(3)$
Curve A	0.242	0.049	1.19
Curve B	0.164	0.049	1.12
Curve C	0.083	0.049	1.03

which neutrons can be scattered from group *i*  and still have an energy  $E_j + \Delta E_j$ . The indicated integration was performed numerically and the anisotropic nature of the elastic scattering was taken into account using the differential cross-section data of Marion *et al. (6)* and Fowler and Cohn *(9).* 

The result of the calculation is shown in Table I. Thus, we obtain values of the fast effect between 1.03 and 1.19. Although these are extreme values, we can calculate a wide range of fast effects that are consistent with different but logical interpretations of the experimental data.

The results quoted are for clean beryllium. After irradiation, the Be becomes poisoned by buildup of Li<sup>6</sup> and He<sup>3</sup> which result from the  $(n, \alpha)$  reaction and its daughter products. Thus, the deleterious effect of the  $(n, \alpha)$  reaction becomes worse with time. This effect is shown in Fig. 2. Thus, if Curve B is correct for the  $(n, 2n)$  cross sections and one assumes no escape of Li, He<sup>3</sup> , or H<sup>3</sup> from the Be, the fast effect becomes zero in 25 years at a flux of 10<sup>14</sup> and is negative thereafter. On the other hand, if Curve C is correct the fast effect is positive for only one year, again assuming no escape of poisons.

Other calculations of the fast effect in Be have been published, *(12, 13, 14, 15, 22).* In general, these authors obtain fast effects of between 1.05 and 1.09 on the basis of assumed *(n,* 2*n)* crosssection curves. As explained above, there is not



FIG. 2. Beryllium poisoning as a function of time due to daughters of the Be  $(n, \alpha)$  reaction.

enough decisive data to make such a choice of the  $(n, 2n)$  cross section and the sensitivity of the results to the curve near threshold appears not to be appreciated. We have recently received a paper from Hafele *(15)* who, using an elegant computational technique, calculates an "optimistic" value of 1.076 for the beryllium fast effect and a "pessimistic" value of 1.051, but we feel that his optimism was somewhat subdued in obtaining his "upper limit" *(n,* 2*n)* cross-section curve and he points out that uncertainties in his results are still large due to the experimental uncertainties.

The fast effect in Be has been measured in Russia *(16)* in four independent experiments. The net result quoted is a fast effect value in clean beryllium of  $1.12\pm0.04$ , but the stated errors in the individual experiments are quite large.

The fast effect in Be holds the possibility of appreciably improving the economy of Be moderated reactors. However, we are critically dependent upon our experimental colleagues for further data in order to determine not only the magnitude of the effect, but also its sign during most of the reactor lifetime.

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