shape during a transient. We would like to add a few comments to this discussion. In general, the transient flux shape is not significantly affected by delayed neutrons if the reactor is small. However, in a reactor which has a dimension twenty times the migration length or more, delayed neutron effects on the transient flux shape begin to be significant. Such dimensions are not unheard of, and in particular, cores of roughly annular geometry such as PWR may have such dimensions around the circumference.

When an asymmetric instantaneous change in material properties is made, the flux shape changes part way toward its asymptotic form in a few prompt neutron lifetimes, but the remaining change takes place in times characteristics of the delay precursors. In this respect, the behavior of the shape is reminiscent of the behavior displayed by the fundamental mode when reactivity is inserted; that is, the reactor first experiences a prompt step change in power level, followed by a slower variation governed by the delayed neutrons. The influence of the delayed neutrons on the transient power shape depends upon how large a part of the flux tilt takes place promptly.

This effect may be seen most easily in a simple model, an initially uniform bare core with one energy group and one group of delayed precursors. At time zero, let a nonuniform perturbation, $\delta\Sigma_a(r)$, be made in the absorption cross section in such a way that the reactor remains critical. After this perturbation, the governing equations are

$$\left[\nabla^2 + B_{m0}^2 + B_{m1}^2 - \beta\,\frac{\nu\Sigma_{f0}}{D_0}\right]\phi + \frac{\lambda C}{D_0} = \frac{1}{D_0\,v}\,\frac{\partial\phi}{\partial t}\,, \tag{1}$$

$$\frac{\partial C}{\partial t} = \beta\nu\Sigma_{f0}\,\phi - \lambda C, \tag{2}$$

where

$$B_{m0}^2 = \frac{\nu\Sigma_{f0} - \Sigma_{a0}}{D_0}\,, \qquad B_{m1}^2 = -\,\frac{\delta\Sigma_a\,(r)}{D_0}\,, \tag{3}$$

and the subscript zero indicates unperturbed quantities.

We may examine the transient behavior of the flux shape by a first-order perturbation theory and a modal expansion. If this is done, the equations governing the expansion coefficients are

$$\begin{aligned}\left[-B_{gk}^2 + B_{m0}^2 - \beta\,\frac{\nu\Sigma_{f0}}{D_0}\right] a_k \\ + \frac{\lambda}{D_0}\,c_k = \frac{1}{D_0 v}\,\frac{d}{dt}\,a_k - \int \psi_k\, B_{m1}^2\,\phi_0\,,\end{aligned} \tag{4}$$

$$(d/dt)c_k = \beta\,\nu\Sigma_{f0}a_k - \lambda\,c_k\,, \tag{5}$$

where a_k is the coefficient of the kth mode in the expansion of the flux perturbation, and c_k is the coefficient of the kth mode in the expansion of the precursor density perturbation. a_0 and c_0 vanish, since we have chosen the disturbance such that the reactor remains critical.

These equations may be readily solved. Figure 1 shows the time dependence of a typical coefficient, a_k, for various values of the parameter $\Delta B_g^2 = B_{gk}^2 - B_{g0}^2$. These curves show that the time dependence of the flux shape consists of a "prompt" jump plus a slow approach to the asymptote with time constant the order of a delayed neutron lifetime. As the buckling difference becomes smaller, the "prompt" jump becomes a smaller part of the whole.

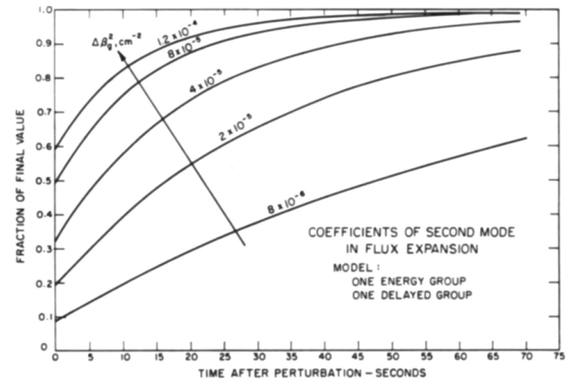

FIG. 1. Time behavior of expansion coefficient.

Solving Eqs. (4) and (5) analytically, one may find that the fractional height of the initial jump is

$$\frac{a_k(0+\epsilon)}{a_k(\infty)} \approx \frac{1}{1 + (\beta k_\infty/M^2\Delta B_{gk}^2)}\,. \tag{6}$$

For a one-dimensional reactor of length L, $\Delta B_{gk}^2 = k(k+2)(\pi^2/L^2)$.

Equation (6) shows that for small graphite-moderated cores of the type examined in reference *2*, in which both M^2 and ΔB_g^2 are large, no substantial effect on the shape is produced by the delayed neutrons. However, for hydrogen-moderated cores with at least one large dimension, the effect of delayed neutrons may be quite significant. For example, if the migration area is about 100 cm², the core length is 500 cm, and $k_\infty \approx 1$; then

$$(\beta k_\infty/M^2\Delta B_{g1}^2) \approx 0.63,$$

and in this case, only about two-thirds of the asymptotic first-harmonic component manifests itself during the prompt jump.

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Analysis of Neutron Flux Data for Accurate Determination of Relaxation Length*

Experiments with exponential piles usually require the measurement of a relaxation length to obtain the essential information about moderators, fuels, or lattices. The relaxa-

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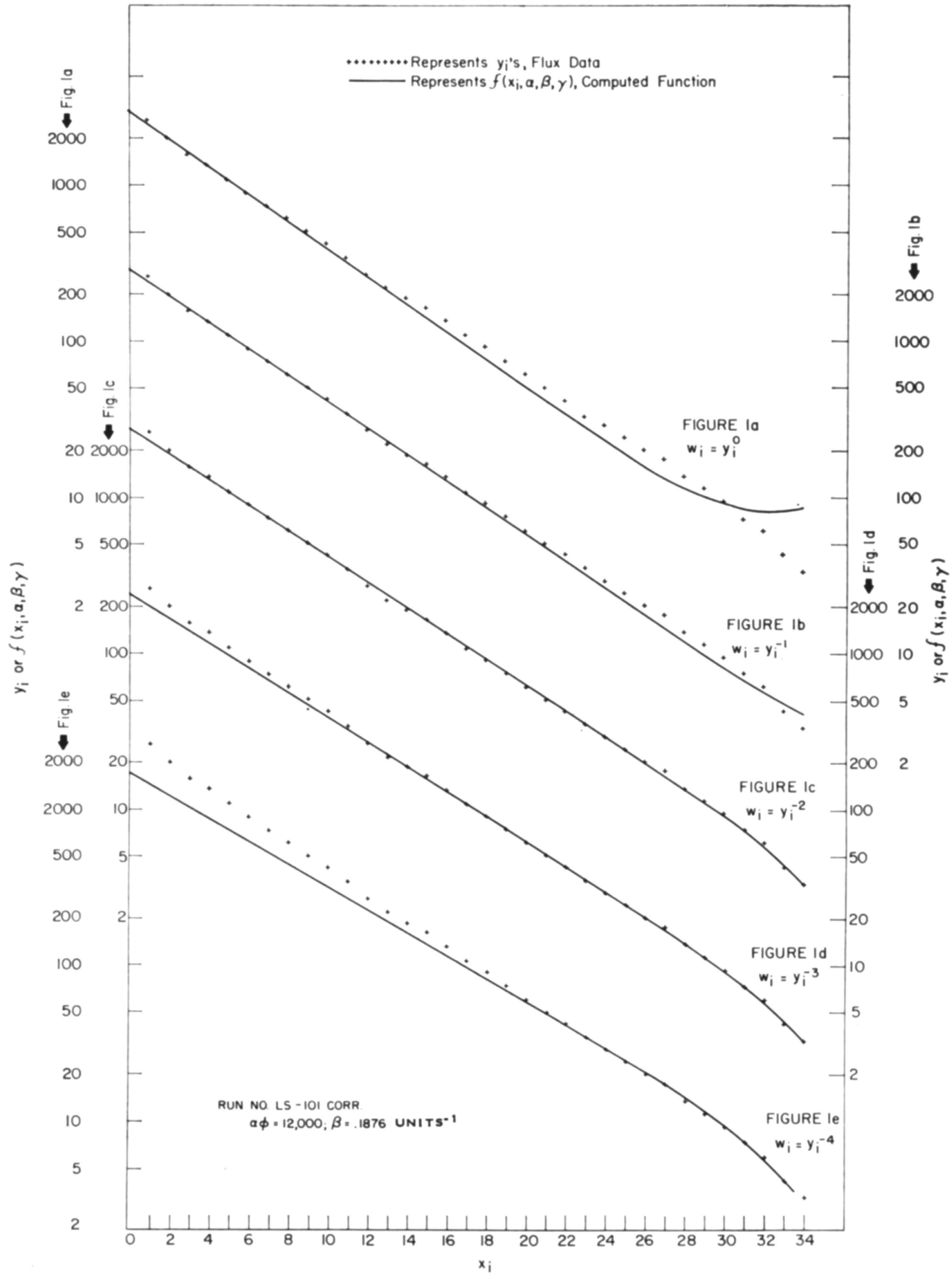

FIG. 1. Plot of flux data y_i against displacement x_i together with five different functions $f(x_i, \alpha, \beta, \gamma)$ resulting from five different methods of weighting the y_i's.

tion length is determined experimentally by flux mapping. Since the data are always subject to statistical errors, it is customary to perform a least squares fit of a function to the data, to obtain the best experimental value of relaxation length. In the process of making a least squares fit, each datum point must be multiplied by an appropriate weighting factor. It is not generally realized, however, that the type of weighting factor depends very strongly on the experimental conditions.

An experiment performed at this laboratory was subject to several types of statistical errors, which could be divided into two groups:

1. Uncertainty of number of counts registered on detector, with each foil counted for the same length of time. This results in a complicated weighting factor, somewhat between $1/y_i$ and $1/y_i^2$ (where y_i is the value of the flux at position x_i).

2. Those types of errors which have the same relative error, and hence should take a weighting factor $1/y_i^2$. These include uncertainty of position of the foil in the pile and also of the foil with respect to the detector, and the uncertainty of the normalization factor of one foil compared to the others.

Because of the above, the proper over-all weighting

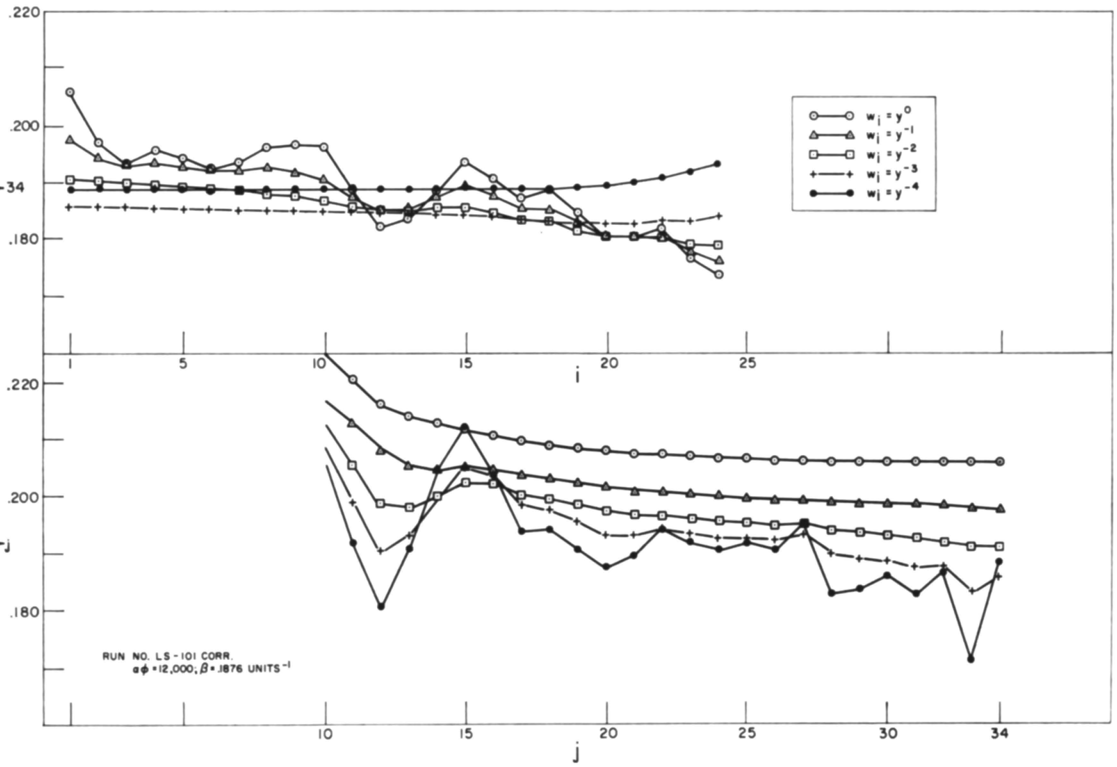

FIG. 2. Slope β as a function of number of data points.

factor was unknown, and an experiment in analysis was tried. Five different weighting factors were used to analyze the same data; the results seem to indicate conclusively that the reciprocal square of magnitude ($1/y_i^2$) is the proper weighting factor, as shown in Fig. 1.

The top curve (Fig. 1a) has each datum point y_i weighted unity. This curve follows the initial points very closely, then gradually ignores the subsequent points. At the bottom end of the curve, there is a complete break away from the data. The next curve (Fig. 1b) has each datum point weighted $1/y_i$ and exhibits the same tendency of ignoring the lower points, but to a smaller degree. The subsequent curve (Fig. 1c) has each datum point weighted $1/y_i^2$ as discussed earlier; this curve fits all the data very well. Beneath this lies the curve (Fig. 1d) which has each datum point weighted $1/y_i^3$; here the curve follows the end points closely but begins to separate from the first points. Finally, the lowest curve (Fig. 1d) has each datum point weighted $1/y_i^4$. It is as bad at the high end as the Fig. 1a curve was at the low end. Considered together then, these curves show that weight unity gives a very bad fit at the low end, weight $1/y$ is less bad but still bad at the low end, weight $1/y^2$ is good throughout, weight $1/y^3$ is poor at the high end, and weight $1/y^4$ is as bad at the high end as weight unity was at the low end.

The analysis just described of the five curves of Fig. 1 can also be made in a different manner, as in Fig. 2. Here the slope β (the main object of the experiment is to determine β, whose reciprocal is the relaxation length) which has been evaluated from the data points, is plotted against range of data points used in the evaluation. The five different curves give the same five ways of weighting the data points y_i as before, *viz*, weight unity, weight $1/y_i$, weight $1/y_i^2$, weight $1/y_i^3$, and weight $1/y_i^4$. The upper group of five curves represents the effect of cutting off initial points of the data; the lower group represents the effect of cutting off the terminal points, as follows: the complete set of data consisted of 34 points. Thus $\beta_{1\to 34}$ indicates the slope as determined using all data points. And $\beta_{i\to 34}$ indicates the slope as determined starting from the point i and using all consecutive points to 34. The running value of i is the abscissa for the upper group of five curves. The symbol $\beta_{1\to j}$ similarly indicates the slope as determined starting from point 1 and using all consecutive points to j. The running value of j is the abcissa for the lower group of five curves.

Analysis of the two groups of five curves goes as follows: if all the data were perfect, there would be only one value of the slope β. Hence, in that case, all five upper curves would coincide to form one horizontal straight line, and the lower five curves would give a similar straight line of equal β value. Because the data are not perfect, it is normal to expect a small point to point variation for the $\beta_{i\to 34}$ and $\beta_{1\to j}$ curves. A relatively large point to point variation would thus indicate that those points have been given too much weight. Consider the two curves for which the data points are weighted y^0 (unity). The upper curve shows large point to point variation, while the lower curve for weight y^0 shows practically no variation at all from point to point (point to point variation refers to random up and down movement of the curves; it does not refer to a systematic slope or curvature). The inescapable conclusion to be drawn from the upper and lower curves for data points weighted y^0 is that the beginning points are weighted too strongly and the last points are ignored, which conclusion is exactly the same as that reached earlier in this paper (see discussion on Fig. 1). Next consider the two curves for which the data are weighted y^{-1}. Here the situation is similar to the weight y^0 curves, but less severe. The upper curve, weight y^{-1}, shows a fair amount of point to point variation and the lower curve shows very little; the beginning points are, therefore, weighted more strongly than the endpoints. The next group of two curves of weight y^{-2} show about the same point to point variation on both the upper and lower curves; this indicates that all points are weighted equally, which is desirable. The two curves weighted y^{-3} exhibit variations just the reverse of the weight y^{-1} curves; i.e., the upper curve shows little variation and the lower curve shows marked variation. Finally, the two curves weighted y^{-4} exhibit variations just the reverse of those of weight y^0. The upper curve of weight y^{-4} shows no point to point variation at all, and the lower curve shows extremely large point to point variations; which indicates that the beginning points are ignored and the end points are weighted too strongly. The y^{-4} weight curve is as bad for the beginning points as the y^0 curve is for the end points, and vice versa for the other ends. Only that curve for which the exponent of the data weight lies midway between the extremes of y^0 and y^{-4}, namely, y^{-2}, shows satisfactory dependence upon all the data points. This analysis and argument then, seems to show that weight y^{-2} is proper for the data points.

An interesting phenomenon occurs at data points 12, 13, and 14. As can be seen from a careful glance at Fig. 1c, these three points are apparently all low, by coincidence. The remarkable thing is the different effect these three points have upon the curves of Fig. 2. The upper curve of weight y^0 shows a distinct trough at points 12 and 13 and the lower curve of weight y^{-4} shows an even more[1] pronounced trough at these points. This very pronounced trough can only mean that the curves containing them weights those points much too strongly, for otherwise two low points would have but slight effect on a slope determined equally from 11 to 22 points. Thus the conclusion reached earlier is again evident here, that weight y^0 is as bad at the front end as weight y^{-4} is bad at the far end. Furthermore the weight y^{-4} upper curve shows no dip at all; likewise the weight y^0 lower curve shows no dip at all. Again, as before, this can only mean that the front end data points of the weight y^{-4} curve are ignored, and also the far end data points of the y^0 curve are ignored. Only the curves of weight y^{-2} shows a slight dip for both the upper and lower; this is what is expected, and furthermore, the exponent 2 being midway between 0 and 4 whose respective weights gave obviously bad results leads again to the conclusion that the weight y^{-2} is indeed the best weighting for the data points.

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[1] The reason that the trough of the lower curve (weight y^{-4}) is more pronounced than the trough of the upper curve (weight y^0) is that the upper trough is generated from 3 data points out of a total of 22, while the lower trough is generated by 3 data points out of 12, i.e., the lower trough is a greater fraction of the total data from which it is generated.