

These can be evaluated readily for the simple geometries by series expansion, in terms of the averages

$$\overline{X^m Y^n} = \frac{1}{V} \int_V X^m Y^n dV$$

with the parameters  $a = \nu/\mu$  and  $\delta = H/H'$ , the latter being related to the reflector savings,  $H' - H$ .

The full expressions for the various quantities involve the four coefficients  $b_2, b_2^*, b_1,$  and  $b_1^*$  of eq. (2). However, the thermal  $b_2$  is by far the largest of the four. For purposes of estimating the effect of the  $Y$  term in the flux it suffices to replace  $B_l(x_j)$  by  $B_l(x_j) - x_j b_2 R_l(x_j)$  everywhere in the equations in (1), the subscript  $j$  referring to the particular isotope whose contribution is under consideration. For large burnups and  $x$  greater than one, the series  $B_l(x) - x b_2 R_l(x) + \frac{1}{2} x^2 b_2^2 S_l(x) - \dots$  must be used to replace  $B_l(x)$ .

The value of  $\delta$  determines what burnup functions must be used. The fractional error in the bare-equivalent theory is given by the ratio  $x b_2 R_l / B_l$ , which is small for  $\delta$  close to one, i.e., small reflector savings. As a rule, the  $B_l$  functions alone may be employed for  $\delta$  between 0.9 and 1, the corrected  $B_l - x b_2 R_l$  being required only for small reactors with  $\delta$  less than 0.9.

The validity of the basic  $B_l$  method is confirmed by calculations for the Yankee Atomic Electric Company reactor, using the specifications listed by Pigford *et al.* (3). This reactor has values of  $\delta_1$  and  $\delta_2$  equal to 0.9336 and 0.9849, respectively, and the  $R_l$  correction is negligible. The core life for the actual nonuniform flux was obtained to be 183 days compared to 176 days found in (3) using computer methods. For the uniform flux case the corresponding values were 316 days and 334 days.

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#### Age in D<sub>2</sub>O-H<sub>2</sub>O Mixtures

The age to indium resonance (1.4 ev) of fission neutrons in various mixtures of D<sub>2</sub>O and H<sub>2</sub>O has been measured by Wade (1). Figure 1 shows this data along with calculated values obtained using the MUFT 4 code (2, 3), which has been modified to include the term  $\partial\lambda/\partial u$  in the Grueling-Goertzel-Amster approximation (4, 5).

MUFT is a multigroup Fourier transform code which solves the Boltzmann equation in slab geometry out to terms in linear anisotropy in the scattering cross sections. The hydrogen, deuterium, and oxygen constants used in this study were prepared by Emmerich (6, 7). In addition to anisotropic scattering, they include such effects as the ( $n, 2n$ ) reaction in deuterium, inelastic scattering by oxygen,

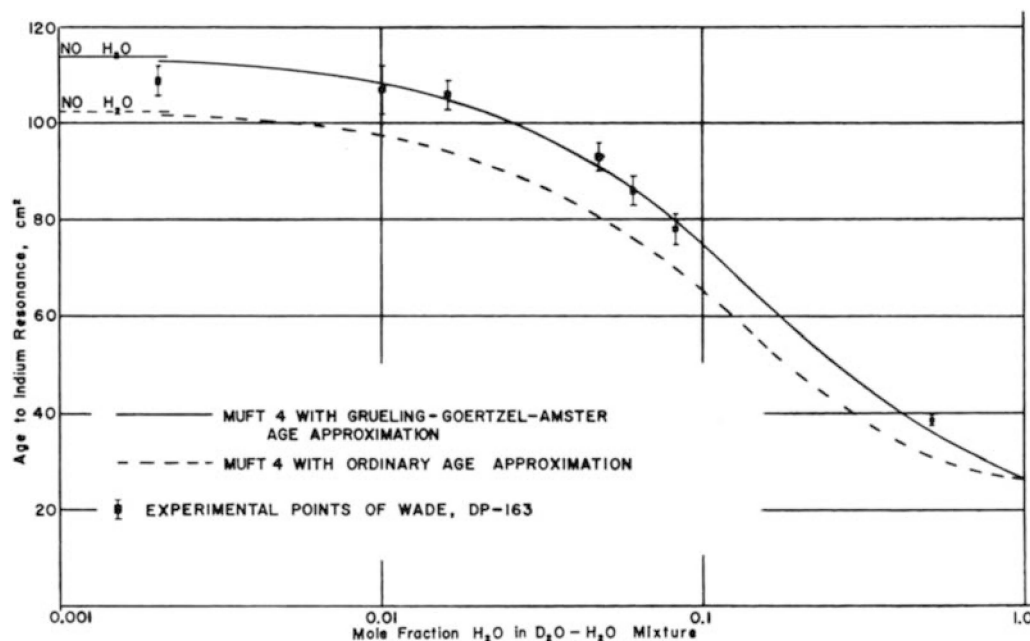


Fig. 1. Comparison between calculated ages in D<sub>2</sub>O-H<sub>2</sub>O mixtures and the experimental data of Wade (1).

and the oxygen ( $n, \alpha$ ) reaction. The values of  $\lambda$  were calculated for deuterium and oxygen from Emmerich's data. The formula for  $\lambda$  in group  $n$  given by Amster (5) simplifies to

$$\lambda_n = \sigma_s^n (T_{00}^2 + T_{01}^2 \overline{\mu_c^n}) / \xi \sigma_s^n$$

where

- $\sigma_s^n$  and  $\xi \sigma_s^n$  are the  $n$ th group zeroth Legendre coefficients of the  $\sigma_s$  and  $\xi \sigma_s$ ,
- $T_{00}^2$  and  $T_{01}^2$  are coefficients defined by Hurwitz and Zweifel (8), having the values 0.4236 and  $-0.8157$ , and
- $\overline{\mu_c^n}$  is the average value of the cosine of the center of mass scattering angle in group  $n$ .

Figure 1 shows the excellent agreement between the MUFT 4 code in its Grenling-Goertzel approximation (modified by the addition of a  $\partial\lambda/\partial u$  term) and Wade's experiments. The dashed curve shows much poorer agreement for the ordinary age theory results.

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### The Slowing-Down Spectrum in a Heterogeneous Reactor

The  $1/E$ -form for the slowing-down neutron spectrum derived from an infinite homogeneous model neglecting resonance capture, is often applied to heterogeneous reactors. This letter is concerned with deviations from the  $1/E$ -form that occur in rod-type reactors as a result of the heterogeneous nature of the fission sources. These deviations will increase with increasing neutron energy and also with increasing lattice pitch. The magnitude of the departure from  $1/E$  has been measured in a ZEEP (1) lattice cell with and without a natural uranium rod and has been

compared with predictions from age diffusion theory. The calculations are similar to those performed by Tralli *et al.* (2) for complete lattices.

The slowing-down spectrum was measured at four energies by measuring the specific activities induced under 0.076-cm thick cadmium in the foils described in Table I.

In the first two detectors in the Table most of the capture occurs near the listed resonance energy, although small contributions occur in higher-energy resonances and in  $1/v$  capture just above the cadmium cutoff at 0.5 ev. The results have not been corrected for these effects. In manganese, however, approximately one-half of the epithermal capture occurs just above the cadmium cutoff. For this reason a method employing resonance self-shielding was developed which permitted the separate measurement of the 337-ev flux. A manganese foil was irradiated inside a manganese cover of 0.013 cm thickness which itself was enclosed in cadmium. The difference in specific activity between the manganese cover and manganese foil is proportional to the 337-ev flux.

Contributions from the higher resonances are not negligible in the case of the uranium detectors. For foils of the thickness used, the contribution from the 6.7-ev level is approximately 43%, from the 21-ev level is 23%, and from the 37-ev level is 15% in a  $1/E$  spectrum.

The measurements were performed in the central cell of ZEEP on a horizontal line passing through the neighboring rods. The rods (1) were immersed in heavy water at a hexagonal spacing of 24.1 cm and consisted of bundles of 19 uranium oxide cylinders of diameter 1.32 cm. The points in Fig. 1 show the experimental flux distributions obtained at the four energies in Table I. The counting errors are all less than 1%, except for the case of the manganese difference which are shown in Fig. 1. Additional errors of approximately 2% for indium, 1% for gold, and 5% for uranium occur due to variations in thickness and composition of the foils and the resulting variations in self-shielding. The upper distributions in Fig. 1 were obtained with the central rod in place, the lower distributions with the rod removed. The "rod-in" and "rod-out" experimental points for a given resonance detector are normalized to the same flux at a reference position in the reflector of ZEEP.

The curves shown in Fig. 1 are predictions of the flux per unit lethargy from age-diffusion theory with 19 energy groups and an infinite reactor. The radial distribution of fissions in the rod based on the measured distribution (3) of thermal neutrons was included in the calculation. Four energy groups were used to describe the fission spectrum. Resonance capture and inelastic scattering in uranium were not included in the calculation. The "rod-in" curves in Fig. 1 were obtained using an equivalent cylindrical cell with zero slope for all fluxes at the cell boundary. The "rod-out" curves in Fig. 1 were calculated by subtracting the fluxes obtained from a single rod in infinite heavy water from the fluxes with the rod in. It was assumed that the fission sources in adjacent rods remained the same when the central rod was withdrawn.

The experimental "rod-in" values for each detector were normalized to the appropriate theoretical "rod-in" curves, and the same normalization factors were applied to the experimental "rod-out" values. In the case of the 337-ev flux, the average value of the flux measured with the rod in was used in the normalization. In the case of In, Au and  $U^{238}$ , the flux distributions were expected to dip