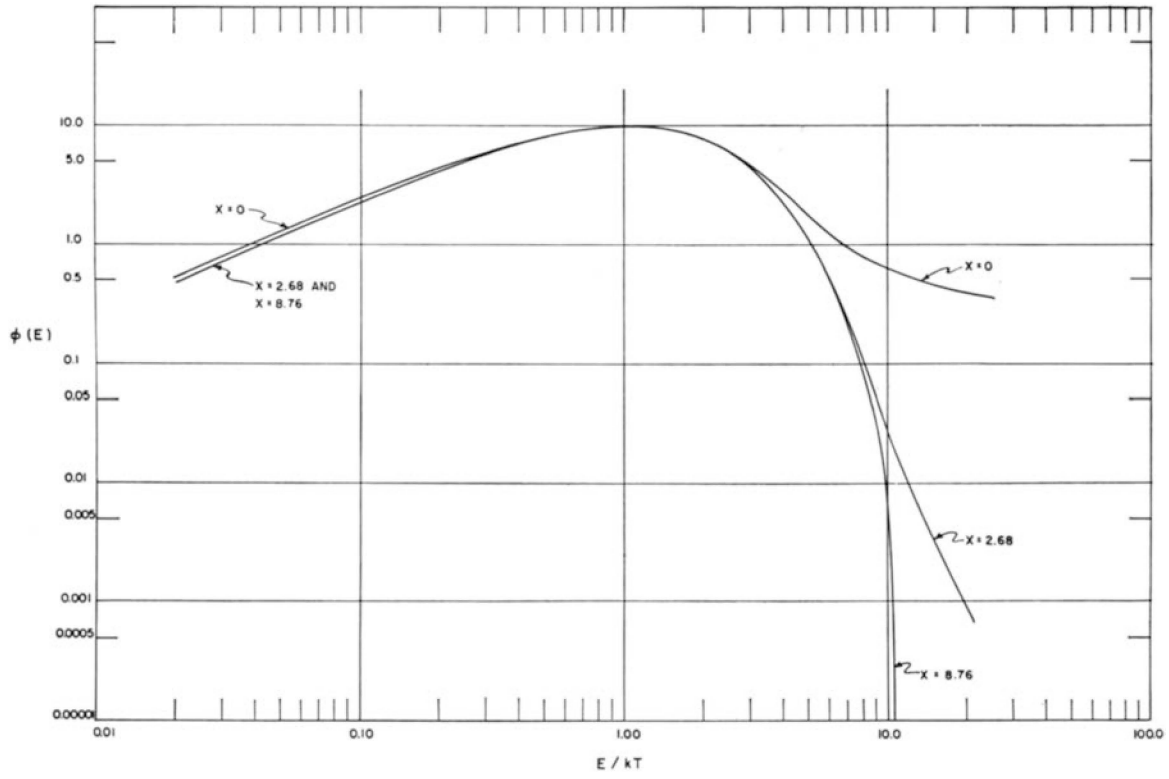

FIG. 2. Thermal flux, as a function of energy, at three different spatial points

Validity of the B_l Method for Fuel Cycle Analysis

The method for burnup analysis of fixed fuel thermal reactors (*1*) makes use of two simplifications, (a) separability of flux in space and time, and (b) representation of flux by the bare-equivalent flux shape. The purpose of this letter is to establish the conditions for validity of these approximations and to extend the analysis by the use of additive corrections to the burnup functions. For large reactors the correction is shown to be unimportant.

The first-order deviation from constant flux shape $\phi(\mathbf{r})$ may be represented by a single harmonic term added to the fundamental, so that

$$\Phi(\mathbf{r}, t) = \phi_F(\mathbf{r})c(t) + \phi_H(\mathbf{r})h(t). \tag{1}$$

It is shown in (*2*) that the B_l theory is corrected to first order for changes in flux shape by using $B_l(x) - yH_l(x)$ wherever $B_l(x)$ occurs, with

$$H_l(x) = \frac{1}{V}\int_V \phi_F{}^l \phi_H \, e^{-\phi_F x}\, dV$$

$$x = \sigma \int_0^t c(t')\, dt'$$

$$y = \sigma \int_0^t h(t')\, dt'.$$

The magnitude of $h(t)$ may be estimated by using the constant power condition together with two group diffusion theory. For a typical enriched reactor, it is found that y is proportional to x^2 and its magnitude is less than $0.1x^2$. Since $H_l(x) < B_l(x)$, the correction for harmonics due to nonseparability of flux is negligible for moderate burnup, i.e., $x < 1$.

The departure from bare-equivalent flux shape near the core-reflector interface is considered by taking the two group fluxes and adjoints in the general form

$$\Phi_i = c(t)[X(\mathbf{r}) + b_i Y(\mathbf{r})] \tag{2}$$

the b_i being constants. The functions $X(\mathbf{r})$ and $Y(\mathbf{r})$ for the infinite slab are cos r and cosh ar, respectively, where $r = \mu z$ and $a = \nu/\mu$. Use of the flux expressions (2) in a treatment parallel to that in Ref. (*1*) leads to equations for the quantities of interest during burnup. In addition to the regular burnup functions

$$B_l(x) = \frac{1}{V}\int_V X^l(\mathbf{r})\, e^{-X(\mathbf{r})x}\, dV$$

the equations involve new burnup functions containing $Y(\mathbf{r})$:

$$R_l(x) = \frac{1}{V}\int_V X^l(\mathbf{r})Y(\mathbf{r})e^{-X(\mathbf{r})x}\, dV$$

$$S_l(x) = \frac{1}{V}\int_V X^l(\mathbf{r})Y^2(\mathbf{r})e^{-X(\mathbf{r})x}\, dV, \quad \text{etc.}$$

These can be evaluated readily for the simple geometries by series expansion, in terms of the averages

$$\overline{X^m Y^n} = \frac{1}{V}\int_V X^m Y^n \, dV$$

with the parameters $a = \nu/\mu$ and $\delta = H/H'$, the latter being related to the reflector savings, $H' - H$.

The full expressions for the various quantities involve the four coefficients b_2, b_2^*, b_1, and b_1^* of eq. (2). However, the thermal b_2 is by far the largest of the four. For purposes of estimating the effect of the Y term in the flux it suffices to replace $B_l(x_j)$ by $B_l(x_j) - x_j b_2 R_l(x_j)$ everywhere in the equations in (*1*), the subscript j referring to the particular isotope whose contribution is under consideration. For large burnups and x greater than one, the series $B_l(x) - xb_2R_l(x) + \frac{1}{2}x^2b_2^2S_l(x) - \cdots$ must be used to replace $B_l(x)$.

The value of δ determines what burnup functions must be used. The fractional error in the bare-equivalent theory is given by the ratio xbR_l/B_l, which is small for δ close to one, i.e., small reflector savings. As a rule, the B_l functions alone may be employed for δ between 0.9 and 1, the corrected $B_l - xbR_l$ being required only for small reactors with δ less than 0.9.

The validity of the basic B_l method is confirmed by calculations for the Yankee Atomic Electric Company reactor, using the specifications listed by Pigford *et al.* (*3*). This reactor has values of δ_r and δ_z equal to 0.9336 and 0.9849, respectively, and the R_l correction is negligible. The core life for the actual nonuniform flux was obtained to be 183 days compared to 176 days found in (*3*) using computer methods. For the uniform flux case the corresponding values were 316 days and 334 days.

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Age in D_2O–H_2O Mixtures

The age to indium resonance (1.4 ev) of fission neutrons in various mixtures of D_2O and H_2O has been measured by Wade (*1*). Figure 1 shows this data along with calculated values obtained using the MUFT 4 code (*2*, *3*), which has been modified to include the term $\partial\lambda/\partial u$ in the Grueling-Goertzel-Amster approximation (*4*, *5*).

MUFT is a multigroup Fourier transform code which solves the Boltzmann equation in slab geometry out to terms in linear anisotropy in the scattering cross sections. The hydrogen, deuterium, and oxygen constants used in this study were prepared by Emmerich (*6*, *7*). In addition to anisotropic scattering, they include such effects as the $(n, 2n)$ reaction in deuterium, inelastic scattering by oxygen,

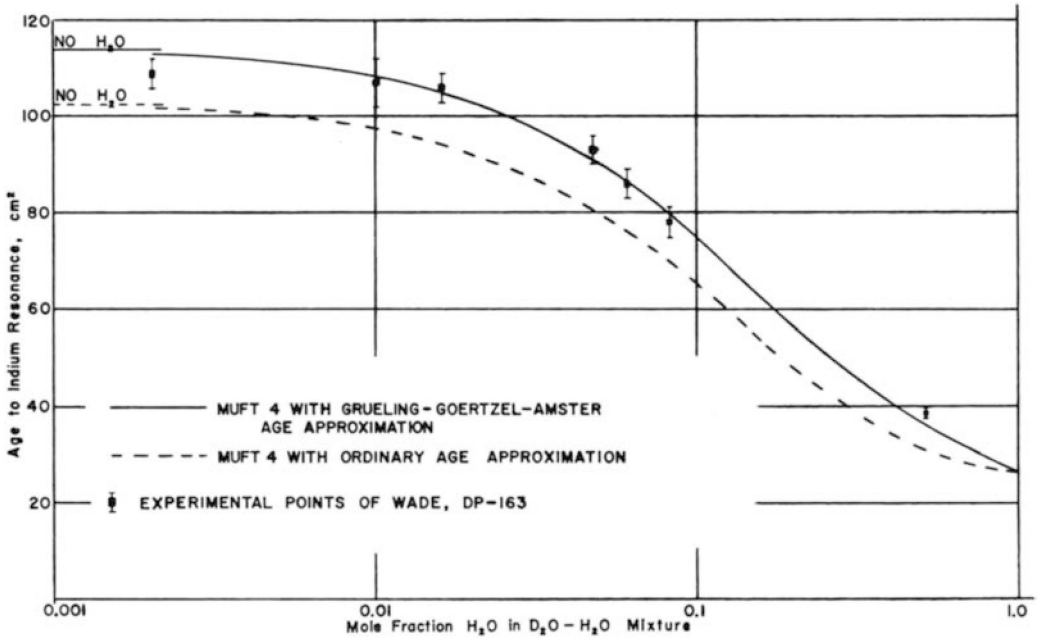

Fig. 1. Comparison between calculated ages in D_2O—H_2O mixtures and the experimental data of Wade (*1*).