causes the mounting plate assembly to move forward toward the ionization chamber. The sealing land engages the O-ring gasket at the mouth of the chamber to effect a seal. Any forward motion of the rod and mounting plate before they are properly positioned is prevented by the ends of the rod that extend into the grooves of the guideways. When in position, the rod can move forward in horizontal slots extending from the groove toward the mouth of the ionization chamber. This further prevents vertical relative motion between the sealing surfaces, before the seal is broken, when the slide assembly is being removed from the counter. To remove the sample the action is merely reversed. The seal is broken by the retracting motion of the mounting plate and disk, and the slide assembly is removed from the counter.

C. L. TRUMAN

Goodyear Atomic Corporation
Portsmouth, Ohio
Received July 2, 1959

On the Use of Dimensionless Variables in Reactor Kinetics

In a recent letter to the Editor (*1*) S. Pearlstein points out the usefulness of the concept of similitude in such reactor problems as criticality or worth of control rod. Dimensionless quantities prove to be quite useful in reactor dynamics as we are going to show in the simple case of kinetics without feedback.

The simple kinetics equations referred to steady initial conditions read:

$$\begin{cases} \dfrac{dn/n_0}{dt/l'} = \dfrac{n}{n_0}(\rho - 1) + \Sigma_i a_i \dfrac{C_i}{C_{io}} \\ \dfrac{dC_i/C_{i0}}{dt/l'} = -\dfrac{l'}{\tau_i}\left(\dfrac{C_i}{C_{i0}} - \dfrac{n}{n_0}\right) \end{cases} \tag{1}$$

where a_i is the relative delayed neutron abundance

$l' = l/\beta k$ is the reduced neutron generation time

and $\rho = \delta k/\beta k$ is the excess reactivity in dollars.

By imposing a step change of reactivity one obtains the classical "inhour" equation which gives reactor period:

$$\rho_0 = 1 + \frac{l'}{T} - \Sigma_i \frac{a_i}{1 + \tau_{i/T}} \tag{2}$$

With a small sinusoidal variation of reactivity and linearization of the kinetics equations (1), one may define a zero power reactor transfer function as the ratio of the amplitudes of power and reactivity variations:

$$W = \left\{1 + j\omega l' - \Sigma_i \frac{a_i}{1 + j\omega\tau_i}\right\}^{-1} \tag{3}$$

Defining gain G and phase shift ρ by the relation $W = Ge^{i\phi}$, one notices that the relations between $Ge^{-i\phi}$ and $j\omega$, ρ_0 and $1/T$ are identical.

TABLE I

	$10^2\,\beta$	$\bar{\tau}$ (sec)	$\bar{\lambda} = \overline{\tau^{-1}}$	$10^2\,\beta\bar{\tau}$	$\bar{\tau}\,\overline{\tau^{-1}}$	$\overline{\tau^2}/\bar{\tau}^2$
U^{235}	0.65	12.75	0.435	8.30	5.55	2.87
Pu^{239}	0.21	14.64	0.39	3.07	5.70	2.48
U^{233}	0.26	17.865	0.30	4.65	5.36	2.52

Equation (2) may be expanded for large reactor period ($\tau_{i/T} \ll 1$):

$$\rho_0 = \frac{l'}{T} + \frac{1}{T}\Sigma_i a_i \tau_i - \frac{1}{T^2}\Sigma_i a_i \tau_i^2 + \cdots \tag{4}$$

Another expansion is obtained for small period ($\tau_{i/T} \gg 1$):

$$\rho_0 = 1 + \frac{l'}{T} - T\Sigma_i a_i/\tau_i \cdots \tag{5}$$

With small neutron generation time ($l'/\bar{\tau} \ll 1$) these two expansions may be written, respectively:

$$\rho_0 \approx \bar{\tau}/T - \left(\frac{\overline{\tau^2}}{\bar{\tau}^2}\right)(\bar{\tau}/T)^2 \tag{6}$$

$$\rho_0 \approx 1 + \left(\frac{l'}{\bar{\tau}}\right)(\bar{\tau}/T) - (\bar{\tau}\,\overline{\tau^{-1}})(T/\bar{\tau}) \tag{7}$$

where we have defined: $\overline{\tau^n} = \Sigma_i a_i \tau_i^n$

The corresponding expressions for phase shift and gain are, respectively:

$$\text{for } \omega\tau_i \ll 1 \begin{cases} -\tan\phi \approx \left(\omega\bar{\tau}\,\dfrac{\overline{\tau^2}}{\bar{\tau}^2}\right)^{-1} \\ G \approx 1/\omega\bar{\tau} \end{cases} \tag{8}$$

$$\text{and for } \omega\tau_i \gg 1 \begin{cases} -\tan\phi \approx \left(\dfrac{l'}{\bar{\tau}}\right)\omega\bar{\tau} + \dfrac{\bar{\tau}\,\overline{\tau^{-1}}}{\omega\bar{\tau}} \\ G \approx \cos\phi = (1 + \tan^2\phi)^{-1/2} \end{cases} \tag{9}$$

In Eqs. (6) and (9) the delayed neutron properties enter only through the dimensionless numbers $\overline{\tau^2}/\bar{\tau}^2$ and $\bar{\tau}\,\overline{\tau^{-1}}$ provided periods are referred to average delayed neutron mean life $\bar{\tau}$. Table I shows that these two dimensionless quantities are quite close for U^{233}, U^{235}, and Pu^{239} though the total delayed neutron fraction may vary from 1 to 3 and the average mean life by about 50% (*2*). Higher moments are also very close.

The asymptotic behavior represented by Eqs. (6) to (9) may be matched by taking only two groups of delayed neutrons (*3*); the corresponding dimensionless constants should be fairly close since they only depend on the two numbers $\bar{\tau}\,\overline{\tau^{-1}}$ and $\overline{\tau^2}/\bar{\tau}^2$. Since Ref. (*3*) shows that this simplified system represents reasonably well the "inhour" equation over a rather large range of excess reactivity, one should expect the same result by plotting ρ (in dollars) *versus* $T/\bar{\tau}$ for different values of the parameter $l/\beta k\bar{\tau}$. Using data from Ref. (*2*) these dimensionless plots prove to be practically the same for the three fissionable isotopes with $l'/\bar{\tau}$ between 10^{-5} and 5×10^{-2} and for ρ from 0 to 1.5 dollars.

Similar results are obtained for reactor transfer function by plotting G and ϕ *versus* $\omega\bar{\tau}$ with $l'/\bar{\tau}$ as parameter. This may be of interest when studying kinetics of a reactor fueled with a mixture of fissionable isotopes. The knowledge

of fuel composition permits definition of proper values of β and $\bar{\tau}$. The variation of reactor period *versus* excess reactivity is then obtained from the dimensionless curves when neutron generation time is known; gain and phase shift variations with frequency are similarly obtained from the proper dimensionless plot. These approximate results could be profitably checked by "prompt burst" neutron decay technique (*4*).

REFERENCES

1. S. PEARLSTEIN, *Nuclear Sci. and Eng.* **5,** 269–270 (1959).

2. G. R. KEEPIN AND T. F. WIMETT, *Nucleonics* 16(10), 86–90 (1958).

3. R. E. SKINNER AND E. R. COHEN, *Nuclear Sci. and Eng.* **5,** 291–298 (1959).

4. G. R. KEEPIN, *Nuclear Sci. and Eng.* **5,** 132–136 (1959).

Department of Mechanical Engineering
Columbia University, New York
GILBERT B. MELESE

Received June 20, 1959