LETTERS TO THE EDITORS

The Transfer Function of a Water-Boiler Reactor

The purpose of this note is to point out an inconsistency in the usual assumptions of the space-independent kinetic equations for a water-boiler reactor as shown in Skinner and Hetrick (*1*).

The specific equations of interest were given as the following:

$$\frac{dP(t)}{dt} + \frac{\beta_{\text{eff}} - \rho(t)}{l} P(t) - \sum_{j=1}^{m} \lambda_j C_j(t) = 0 \qquad (1.1)$$

$$\rho(t) = \rho_1(t) + \alpha T(t) + \phi V(t) \qquad (1.5)$$

For simplicity, we ignore the delayed neutron terms, and find

$$\frac{dP(t)}{dt} = -\frac{\beta_{\text{eff}} - \rho(t)}{l} P(t)$$

Strictly speaking, this equation is applicable only to reactors operating at fixed values of ρ_1, T, and V. The values may be changed from run to run, but not during the run. If P is a function of ρ, T, and V as Skinner and Hetrick have found, then with full mathematical rigor we may write

$$\frac{dP(t)}{dt} = \frac{\partial P}{\partial t}\Big)_{V,T} + \frac{\partial P}{\partial V}\Big)\dot{V}_{t,T} + \frac{\partial P}{\partial T}\Big)\dot{T}_{t,V} \qquad (1)$$

We now argue that by definition, and by actual experimental practice

$$-\frac{\beta_{\text{eff}} - \rho(t)}{l} P(t) \equiv \frac{\partial P}{\partial t}\Big)_{V,T}$$

We therefore claim that it is incorrect to assume that Eq. (1.1) is complete without specifying the conditions under which the additional terms may be neglected. It is not mathematically sufficient to argue that because Eq. (1.5) does not contain terms in $\dot{V}$, or $\dot{T}$, Eq. (1.1) should not contain terms in $\dot{V}$. In a derivation proceeding from fundamental principles, the term in $\dot{V}$ does not enter Eq. (1.1) through Eq. (1.5).

It is believed that the application of Eq. (1) is extremely important for a wide class of reactor kinetic problems (*2*). Furthermore, some of the statements of Skinner and Hetrick should be reviewed in the light of the approximations which they have made. For example, the statement on page 591, "From a pragmatic viewpoint, the range of validity of the space-independent kinetic equations may be extended by regarding the paramenters K, α, γ, G, ϕ, and σ as functions of the equilibrium power, with all other external experimental parameters constant, as measured from the experimental transfer function." is strictly true only for very small values of both $(\partial P/\partial V)_{t,T}\dot{V}$ and $(\partial P/\partial T)_{t,V}\dot{T}$.

Another way to arrive at Eq. (1) is through the dependence of "power" on the "effective" neutron generation time as follows:

$$\dot{P} = \frac{\partial P}{\partial t}\Big)_{l} + \frac{\partial P}{\partial l}\Big)\dot{l} = \frac{\partial P}{\partial t}\Big)_{l} + \frac{\partial P}{\partial l}\left[\frac{\partial l}{\partial V}\Big|_{T,t}\dot{V} + \frac{\partial l}{\partial T}\Big|_{V,t}\dot{T} + \frac{\partial l}{\partial t}\Big|_{V,T}\right]$$

$$= \frac{\partial P}{\partial t} + \frac{\partial P}{\partial V}\dot{V} + \frac{\partial P}{\partial T}\dot{T}$$

if $\partial l/\partial t = 0$.

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Multigroup Diffusion Calculation of k_∞ for D_2O Lattices[1]

In the usual method of calculating the criticality of lattice reactors the actual lattice cell is replaced by a cylindrical cell of the same area. The infinite multiplication constant is then calculated from the four-factor formula, $k_\infty = \eta\epsilon pf$. When the factors are calculated using standard recipes (*1*), it is usually found that the values of k_∞ obtained are greater than experimental values. To correct for this, various semi-empirical formulae have been devised to modify the values of one or more of the four factors (*2*).

In an attempt to calculate k_∞ from basic data, we have developed a multigroup procedure for a single reactor cell. This method yields not only k_∞ but also a consistent set of the four factors. The definitions of ϵ and p are different from those usually employed. In particular, ϵ is defined as the ratio of the total neutron production to the thermal production and p is defined as the ratio of the thermal absorption to the total absorption in the cell. However, since the definitions of η and f are the usual ones the product ϵp conforms to the usual definition, $k_\infty/\eta f$.

[1] Work performed under Subcontract 1295 of U. S. Atomic Energy Commission Contract AT(38-1)-193, with Sargent & Lundy Engineers.

TABLE I

COMPARISON OF MULTIGROUP AND STANDARD RESULTS FOR A 1.50-IN. DIAMETER NATURAL URANIUM ROD IN D_2O LATTICES

(Rods were clad with 0.102 cm of Al; H/D = 0.0024.)

	Pitch = 6.00 in.		Pitch = 7.25 in.		Pitch = 9.00 in.		Pitch = 12.00 in.	
	Mult.	Stand.	Mult.	Stand.	Mult.	Stand.	Mult.	Stand.
η	1.315	1.315	1.315	1.315	1.315	1.315	1.315	1.315
ϵ	1.092	1.0459	1.072	1.0459	1.059	1.0459	1.051	1.0459
p	0.829	0.906	0.872	0.936	0.902	0.958	0.923	0.970
f	0.985	0.985	0.980	0.980	0.971	0.971	0.949	0.949
k_∞	1.172	1.227	1.204	1.262	1.220	1.279	1.212	1.266

TABLE II

RELATIVE NEUTRON FLUX PER UNIT LETHARGY AT THE SURFACE OF A 1.50-IN. DIAMETER NATURAL URANIUM ROD IN D_2O LATTICES

Group	Lethargy Range	Pitch (in.) 6.00	7.25	9.00	12.00
3	2.25–6.75	2.031	2.457	3.282	4.829
4	6.75–9.8	1.370	1.427	1.603	2.012
5	9.8–12.0	1.254	1.259	1.313	1.469
6	12.0–12.7	1.164	1.160	1.181	1.258
7	12.7–13.633	1.107	1.103	1.112	1.147
8	13.633–14.5	1.000	1.000	1.000	1.000

The results of any multigroup calculation depend on the group structure and the methods employed to obtain group-averaged cross sections. The results given below are for 1.50-in. diameter natural uranium rods in a square array in D_2O. The rods were clad in 0.102-cm thick aluminum tubes. In preparing the cross sections for the uranium both Doppler broadening and energy self-shielding were taken into account. If a $1/E$ neutron spectrum is assumed, our group-averaged cross sections yield the same resonance integral as predicted by the Hellstrand formula (*3*) for the energy region below about 100 kev. Further details of the treatment of the U^{238} resonances and the calculation of the group-averaged cross sections will be published later.

The purpose of this letter is to give a preliminary report of the following interesting results:

1. The values of k_∞ obtained from the multigroup calculations are consistently lower than those given by standard procedures; see Table I. The difference is about 4.5 per cent. Since the multigroup values of η and f are in agreement with the standard values, the variation of the product ϵp is the same as that of k_∞.

2. The values of $1 - p$, the resonance absorption probability, obtained from the multigroup calculations are consistently higher than those obtained from standard procedures, the per cent difference decreasing with decreasing pitch. This is due to the departure of the neutron spectrum from the $1/E$ spectrum assumed in the standard procedures. This is discussed in item 3 below.

3. The multigroup calculations show that the neutron spectrum at the rod surface is not $1/E$. For a given pitch the spectrum departs from $1/E$ increasingly with increasing energy. In addition, the neutron spectrum is found to come closer to the $1/E$ spectrum as the pitch decreases. These results are summarized in Table II, in which the relative neutron flux per unit lethargy at the rod surface is tabulated. Since the flux per unit lethargy is not constant, the spectrum is not $1/E$.

The departure of the spectrum from $1/E$ results in higher values for the high energy absorption than those predicted by the standard procedure, partly explaining the lower values of p. Another reason for the lower p values is the inclusion of the nonthermal U^{235} absorptions.

4. The multigroup calculations yield values of ϵ which increase with decreasing pitch. The increase in the value of ϵ with decreasing pitch was found to be primarily due to the nonthermal fissions in U^{235}, which are not usually included in the Spinrad formulation (*4*). When the nonthermal fission cross section in U^{235} was set equal to zero the multigroup calculations yielded values of ϵ which ranged from 1.045 for the smallest pitch to 1.036 for the largest pitch. This may be due to a diffusion theory overestimate of rod-rod interaction. The constant value 1.0459 for ϵ listed under the standard results in Table I was obtained using a Spinrad formulation.

5. The results of the multigroup calculations were used to calculate values of the neutron age to thermal, τ, and the thermal neutron diffusion area, L^2. Values of the buckling, B^2, were then calculated from the criticality relation, $k_\infty = (1 + L^2B^2)(1 + \tau B^2)$. Table III gives the comparison of the calculated values with the experimental results from the North American Aviation (NAA) exponential experiments (*5*) and the Chalk River critical experiments (*6*). It is seen that the multigroup results are in good agreement with all the experimental results.

Calculations are now in progress comparing the results of the multigroup procedure with experimental values of B^2, k_∞, τ, and L^2 for a variety of cases.

TABLE III

COMPARISON OF MULTIGROUP CALCULATIONS OF BUCKLING (B^2) WITH EXPERIMENTAL VALUES FOR 1.50-IN. DIAMETER NATURAL URANIUM RODS IN A SQUARE ARRAY IN D_2O (Rods were clad with 0.102 cm Al)[a]

Pitch (in.)	B^2 (m^{-2}) Calculated	Measured	Per Cent difference	Installation	Facility
6.00	8.13	8.65	−6.0	NAA	Exponential
6.93	7.80	7.96 ±0.076	−2.0	Chalk River	Critical
7.25	7.59	7.94	−4.4	NAA	Exponential
7.75	7.135	7.236 ±0.069	−1.4	Chalk River	Critical
9.00	6.00	6.29	−4.6	NAA	Exponential
12.00	3.63	3.69	−1.6	NAA	Exponential

[a] D_2O purity:
NAA, H/D = 0.0024
Chalk River, H/D = 0.0028.

Note Added in Proof: Recent detailed analysis of the multigroup diffusion calculations indicates that the good agreement between calculation and experiment is partly fortuitous. The effect of the diffusion approximation as applied to the D_2O is to underestimate both the ratio of U^{238} fissions to thermal fissions and the resonance escape probability. Further study of these effects is continuing.

REFERENCES

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Corrosion of Irradiated Uranium Alloys

The effect of nuclear radiation on some corrosion-resistant uranium alloys has been described in previous papers (*1*, *2*). Essentially, we learned that small radiation dosages effectively destroyed the corrosion resistance of metastable uranium alloys heat-treated for corrosion resistance.

Accordingly, the fuel for the EBWR, U-5% Zr-1½% Nb, was heat-treated for dimensional stability under irradiation. This treatment comprised heating at 825°C, quenching at 640°C, holding at 640°C for 23 hr, and air cooling. In this condition the fuel material has a corrosion rate[1] at 260°C in initially pure water of 9470 mg/cm²/day. For comparison, the corrosion rate for unalloyed uranium under the same conditions is about 64,000 mg/cm²/day.

At a scheduled shutdown of the EBWR a subassembly was removed and sections taken from one of the plates for corrosion testing. Burnup varied with position in the plate and was determined by radiochemical analysis. The sections were exposed to initially pure water at 260°C–270°C for aproximately one day. Results are summarized in Table I.

The results are interesting and surprising. Because of the nature of the work, the results reported here (as well as most of those in previous studies) are necessarily based on one or two samples at each level of burn-up. However, the trend observed appears to be definite enough to warrant publication. At the present time we have no explanation for the improvement in corrosion resistance or the apparent minimum in corrosion rate as a function of burn-up.

This study is continuing and results will be published as they become available.

The samples and burnup data were furnished by C. F. Reinke of the Metallurgy Division.

TABLE I

Corrosion of Irradiated U-5% Zr-1½% Nb at 260°–270°C as a Function of Burnup

Sample No.	Burnup, Total a/o	Corrosion Rate, mg/cm²/day
	0.000	9470
1	0.005	2500
15	0.009	2780
13	0.017	2310
12	0.024	1880
10	0.088	1890
6	0.140	5160

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Measurements of the Transport Mean Free Path of Thermal Neutrons in Beryllium as a Function of Temperature

In order to calculate temperature effects on reactors it is important to know the variation of the transport mean free path of thermal neutrons with temperature. K. S. Singwi and L. S. Kothari (*1*) have calculated this variation for different crystalline materials. In an attempt to confirm these calculations experimentally, we investigated the variation with temperature of the transport mean free path, λ_{tr}, of thermal neutrons in beryllium using the pulsed-neutron technique (*2*). The decay constant, λ, of the fundamental mode of the thermal neutron density in a beryllium assembly 20 by 20⅛ by 20⅛ in. (buckling, $B^2 = 1.05 \times 10^{-2}$ cm^{-2}) was measured for 10 different temperatures of the beryllium, ranging from −46° to +511°C.

The reciprocal thermal-neutron lifetime, $\lambda_a = 288 \pm 60$ sec^{-1}, and the room-temperature diffusion cooling constant, $C = 1.1 \pm 1$ cm², for this beryllium had been previously measured (*3*). The relation between the decay constant λ

[1] The rates discussed in this letter are based on original exposed area. True rates would therefore be higher but the quoted rates are valid for purposes of comparison.

[2] Operated for United States Atomic Energy Commission by the University of Chicago.