

## REFERENCES

1. J. YVON, *J. Nuclear Energy* **4**, 305 (1957).
2. L. SCHWARTZ, "Théorie des Distributions," Herman, Paris, 1950, 1951.
3. H. KOENIG, *Math. Nachr. Berlin* **9**, 129 (1953).
4. W. GUTTINGER, *Progr. Theoret. Phys. Kyoto* **13**, 612 (1955).
5. A. SAUER, ORNL-CF-58-129 (1958).
6. B. DAVISON, "Neutron Transport Theory," p. 172. Oxford, London and New York, 1957.

Nuclear Division  
Combustion Engineering, Inc.  
Windsor, Connecticut

D. W. DRAWBAUGH  
L. C. NODERER

Received April 10, 1959.

### The Age of $U^{233}$ Fission Neutrons in Water

The age to indium resonance energy (1.4 ev) of  $U^{233}$  fission neutrons slowing down in water has been calculated with the SLAG code (1) on the University of Michigan IBM-650. The fission spectrum used was measured at Oak Ridge (2) with the low energy portion ( $E < 1.3$  Mev) taken to be the ( $U^{235}$ ) Watt spectrum (3). This spectrum, and the

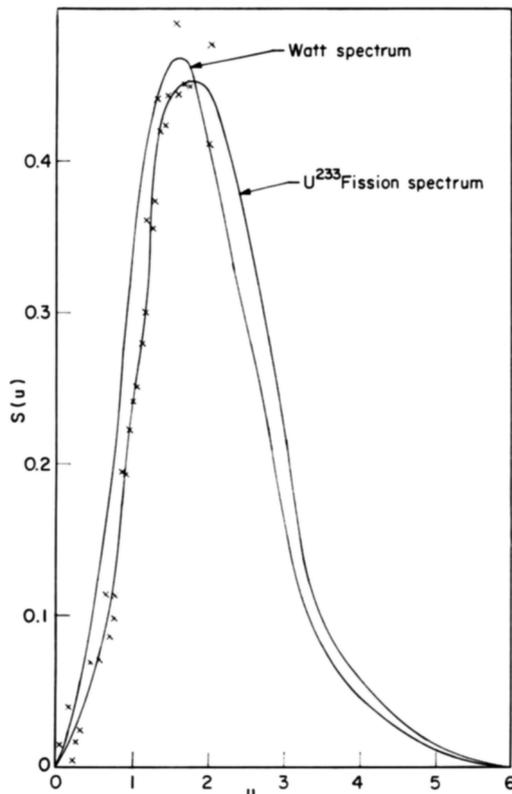


FIG. I. Fission neutron spectrum of  $U^{233}$  and  $U^{235}$ . The  $U^{235}$  spectrum is the Watt spectrum given by  $\chi(E) = 0.484 e^{-E} \sinh \sqrt{2E}$ .

Watt spectrum, both normalized to unit area, are given in Fig. I as a function of  $u = \ln(10^7/E)$  where  $E$  is in electron-volts. The crosses indicate the experimental points, and the smooth curve has been drawn by eye.

The result for  $\tau$  (1.4 ev) is  $23.0 \pm 3$  cm<sup>2</sup> compared to a value of 25.3 cm<sup>2</sup> from the Watt spectrum. The error limits were obtained from calculations for curves passing through the maximum and minimum error points of the measured spectrum. In order to test the sensitivity to the fit to the experimental points, a calculation was made for a curve generated by connecting adjacent experimental points, with no change being observed in the age. Finally, a calculation was made for the measured fission spectrum of  $U^{235}$  reported in reference 1, which differs slightly from the Watt spectrum, in order to test the possibility of consistent errors in the fission spectrum measurements. However, this spectrum led to a value of  $\tau$  identical with that given by the Watt spectrum.

The results of these calculations indicate that the age of  $U^{233}$  fission neutrons is probably about 8 or 9 per cent lower than the age of  $U^{235}$  fission neutrons, a fact which has important implications in the measurement of  $\eta$  of  $U^{233}$  by critical experimental techniques, such as are now being used at ORNL (4). In addition, it indicates that the losses to fast leakage from a  $U^{233}$  system will be somewhat less than those from a  $U^{235}$  system, which improves the possibility of breeding. It is clear, however, that an age experiment with  $U^{233}$  fission neutrons must be performed because of the fairly large error limits on the measured spectrum which lead to the error limits of  $\pm 3$  cm<sup>2</sup> in the age.

The authors wish to thank the members of the statistical and computing laboratory at the University of Michigan both for the use of their IBM machines and for assistance in running the calculations. The curve was drawn by R. J. Mack.

## REFERENCES

1. C. D. PETRIE, KAPL-1789 (1957).
2. K. M. HENRY, ORNL 2389, p. 111 (1957).
3. B. E. WATT, *Phys. Rev.* **87**, 1037 (1952).
4. E. P. BLIZARD, private communication (1959).

University of Michigan  
Ann Arbor, Michigan

MARTIN FABER  
P. F. ZWEIFEL

Received March 3, 1959

### Note on Position-Dependent Spectra

Near boundaries between dissimilar media ( $A$ ,  $B$ , etc.), the spatial behavior of the thermal neutron flux is predicted incorrectly by normal diffusion theory, in which energy averages of the constants are taken over the asymptotic spectra of  $A$ ,  $B$ , etc. To some extent transport corrections are necessary but the predominant effect is that due to a continuous change of spectrum with position in going from one medium to another.

In particular, the observed peaking from a slab water gap immersed in a multiplying medium is always higher than that predicted by normal diffusion theory. On the other hand, the first correction term arising from a one-

TABLE I  
COMPARISON OF EXPERIMENTAL AND CALCULATED  
RELAXATION LENGTHS

Lattice	$L_{\text{exp}}$	$L_{\text{SOFOCATE}}$	$L_{\gamma}$
A	1.76	2.28	1.74
B	2.14	2.68	2.22
C	2.35	3.15	2.46
D	1.10	1.42	1.015

velocity  $P_3$  calculation is negative and thereby reduces the peaking calculated in diffusion theory.

Recently a series of activation experiments have been performed (1) in which the spatial transients arising from hydrogenous slab gaps have been studied in detail in a number of different media. Within the experimental uncertainty, the transients followed a single exponential with a relaxation length *not* characteristic of the asymptotic spectrum of the medium. A reasonable model that would lead to this experimental result is that of a position-dependent spectrum near the gap given by

$$\phi^c(E, X) = \phi_{\infty}^c(E) + \gamma(E)e^{-(X-a)/L_{\gamma}}$$

where

$\phi_{\infty}^c(E)$  = asymptotic spectrum of the core taken as the SOFOCATE (2) spectrum for the particular medium,

$\gamma(E)$  = position-independent spectrum of the spatial transient,

$L_{\gamma}^2 = \langle D^c(E) \rangle / \langle \Sigma_a^c(E) \rangle$  averages to be taken over  $\gamma(E)$ ,  
 $a$  = half thickness of the gap.

Assuming that the epithermal flux does not change in the gap region ( $1\frac{1}{2}$  group model) and that the gap spectrum is also a superposition of spatially independent spectra one writes:

$$\phi^g(E, X) = \phi_{\infty}^g(E) + \beta(E) \cosh X/L_{\beta}$$

where the terms are similar to those above. Then, equating flux and current at the interface,  $x = a$ , at each energy:

$$\gamma(E) = \frac{\phi_{\infty}^g(E) - \phi_{\infty}^c(E)}{1 + \frac{D^c(E) L_{\beta}}{D^g(E) L_{\gamma}} \coth a/L_{\beta}} \quad (1)$$

and

$$\beta(E) = -\gamma(E) \frac{D^c(E) L_{\beta}}{D^g(E) L_{\gamma}} \operatorname{csch} a/L_{\beta} \quad (2)$$

The  $1\frac{1}{2}$  group model requires that the two asymptotic spectra are to be normalized in the  $1/E$  tail.

These expressions yield a method of calculating the peaking and relaxation length of a transient generated by a slab gap. It should be noted that a first choice of  $L_{\gamma}$ ,  $L_{\beta}$  can be obtained by using

$$\gamma(E) \sim \beta(E) \sim \phi_{\infty}^g(E) - \phi_{\infty}^c(E) \sim \phi_{\max}(kT_{\text{eff}})$$

since the difference between the normalized SOFOCATE spectra is approximately a Maxwellian at the effective temperature of the gap and the denominator of Eq. (1) is slowly varying in energy. This latter point is consistent with the fact that the experimental relaxation length appeared to be independent (insensitive) to gap thickness.  $L$  values calculated with this choice of spectrum show much better agreement with experiment than those calculated using  $\phi_{\infty}^c(E)$ .

If one defines the peaking as the activity measured at the gap-core interface to that observed in the asymptotic region, the model above gives an unambiguous method of peaking computation for any foil cross section, namely:

$$\text{peaking} = 1 + \frac{\int_0^{\infty} \sigma_{\text{foil}}(E) \gamma(E) dE}{\int_0^{\infty} \sigma_{\text{foil}}(E) \phi_{\infty}^c(E) dE}$$

In the case of slab thermal poisons, exponential relaxation of the transient was also observed experimentally in a number of cases. Using an energy-dependent transmission boundary condition, one again might assume space-energy separability of the transient and write

$$\gamma_{\text{poison}}(E) = \frac{\phi_{\infty}^c(E)}{1 + \frac{2D_c(E)}{L_{\gamma}} \frac{1 + T(E)}{1 - T(E)}}$$

where  $T(E)$  = energy-dependent transmission of the poison slab.

Extension of this model to many regions is immediate via the principle of superposition of spectra. It should be pointed out that knowing  $\phi(E, X)$  allows calculation of  $\sigma(X)$  for multigroup calculations.

#### REFERENCES

1. W. B. WRIGHT AND F. FEINER, Paper 3-7, ANS Meeting at Detroit, Michigan, December 8-10, 1958.
2. H. AMSTER, "Cross Sections in the SOFOCATE Code: Second Deck," WAPD-TM-67 (1957).

General Electric Company

Knolls Atomic Power Laboratory

Schenectady, New York

Received April 13, 1959

W. B. WRIGHT

F. FEINER