2. G. GOERTZEL, Nuclear Development Corporation of America, AND D. SELENGUT, General Electric Company, devised a theory for hydrogen moderated reactors which takes account of the fission spectrum and the slowing down of neutrons by heavy elements. See report TAB-53.

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A Contribution to Black Control Rod Worth

In a recent paper, Arnold (*1*) makes a useful suggestion enabling a calculation of control rod worth entirely from a perturbation analysis. An equivalent cross section, $\overline{\Sigma^{\text{rod}}}$, is defined for each group that will lead to the same reactor averaged cross section in the original flux ϕ as the actual property perturbation P in the new flux ϕ'. For example, in the thermal group, it is sufficient that

$$\frac{P_{22}\,\phi_2' + P_{21}\phi_1'}{\int \phi_2'\,dv} = \frac{\overline{\Sigma_2^{\text{rod}}\phi_2}}{\int \phi_2\,dv} \tag{1}$$

where P_{22} represents the perturbation in slow leakage and absorption and P_{21} is the perturbation in the source term from the fast group. The integrals are over the reactor volume.

An equivalent perturbation $\bar{P}_2$, however, can be defined for the terms such as P_{22} and P_{21}, that will lead to the same asymptotic period in the reactor. According to the method of Pigford *et al.* (*2*), it is sufficient that

$$\frac{\phi_2^* P_{22}\,\phi_2' + \phi_2^* P_{21}\,\phi_1'}{\int [(\phi_1^*\phi_1'/V_1) + (\phi_2^*\phi_2'/V_2)]\,dv} = \frac{\phi_2^*\bar{P}_2\phi_2}{\int [(\phi_1^*\phi_1/V_1) + (\phi_2^*\phi_2/V_2)]\,dv} \tag{2}$$

V represents the group speeds. The adjoint flux, ϕ_2^*, cancels in the numerator of Eq. (2) and, on defining

$$F_2 = \frac{\int \phi_2'\,dv \int [(\phi_1^*\phi_1/V_1) + (\phi_2^*\phi_2/V_2)]\,dv}{\int \phi_2\,dv \int [(\phi_1^*\phi_1'/V_1) + (\phi_2^*\phi_2'/V_2)]\,dv} \tag{3}$$

we have

$$\bar{P}_2 = F_2\overline{\Sigma_2^{\text{rod}}}. \tag{4}$$

The F ratios have the following significance: the reactivity is affected by the flux change as well as by the property perturbation. For example, the leakage out of a control rod

perturbed reactor is likely to be increased [e.g., see (*3*), chapter 11.10]. Thus, the effective perturbation in each group is larger than the effective cross section by the ratio F. This ratio depends on the speed group; the thermal flux may be more severely changed than the fast flux.

Arnold takes $\bar{P}_2$ to be equal to $\overline{\Sigma_2^{\text{rod}}}$, the accuracy evidently depending on the value of F_2. We have calculated an F by one group diffusion theory for two simple cases, to investigate the order of magnitude:

(a) One region slab with central black slab. For values of control slab to reactor extrapolated thicknesses

$(2b/2H)$	0.250	0.125	0.050	⋯	0
(F)	1.46	1.31	1.22	⋯	1.18.

(b) One region coaxially rodded cylinder. For values of the ratio of inner to outer extrapolated radii

(R'/R)	0.250	0.125	0.050	⋯	0
(F)	1.31	1.16	1.06	⋯	1.

Neglecting F in these one region cases corresponds to neglecting the increased leakage and underestimates the reactivity worth of the rod.

In the particular example given by Arnold to calculate $\overline{\Sigma_2^{\text{rod}}}$, we consider a semi-infinite absorber adjacent to a black half-space and with a uniform source of thermal neutrons. Diffusion theory is applied to obtain the neutron interface current, using the extrapolation distance devised from the Milne problem. This is exact only in the limit of the Milne problem itself (pure scatterer with plane source at infinity). Davison however has given a transport solution to the present problem in terms of the solution to the Milne problem [(*4*), pp. 79–81]. Introducing Placzek's numerical solution to the Milne problem (*5*) and integrating numerically, the expression for the interface current is

$$j_{ir}(0) = (S/c\kappa)[1 - 0.7104\ (\kappa/\Sigma)] \tag{5}$$

where S is the volume source strength, c the multiplicity $(\nu\Sigma_f + \Sigma_s)/(\Sigma_a + \Sigma_s)$, Σ the total cross section and κ the reciprocal diffusion length.

The resulting equivalent cross section for a strip of thickness $2b$ is now

$$\overline{\Sigma_2^{\text{rod}}} = \frac{\Sigma_2^a}{bc\kappa}\left[1 - 0.7104\,\frac{\kappa}{\Sigma}\right]. \tag{6}$$

This can be compared with the diffusion expression given by Arnold

$$\overline{\Sigma_2^{\text{rod}'}} = \frac{\Sigma_2^a}{b}\left(\frac{1}{\kappa + 2.13\,\Sigma_2^a}\right). \tag{6'}$$

For regions with negligible absorption, these two expressions are identical. A calculation with typical values for an enriched uranium/D_2O core (c =0.97), shows the diffusion expression to overestimate the cross section (and hence the reactivity worth of the rod) by a ratio 1.05 in this example. On the other hand, in strong absorbers where $c < 1$, the diffusion expression will underestimate. In this circumstance, it will be advantageous to use the simple transport expression derived here if the F ratios are to be taken as unity.

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Absence of Irradiation Growth in Alpha-Uranium above 430°C

The theory of Seigle and Opinsky (*1*) explains growth of alpha-uranium on irradiation by anisotropy of diffusion of interstitial uranium atoms and of vacancies in orthorhombic lattice. To show the usefulness of and to provide a possibility of testing this theory, the

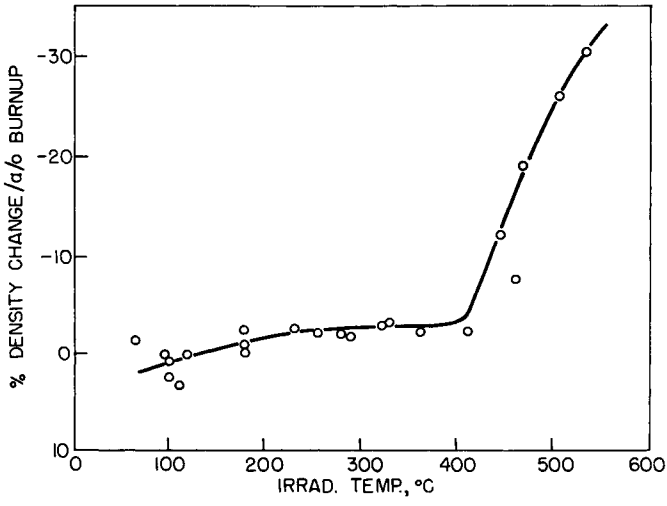

FIG. 1. Density change per atomic per cent burnup in uranium as a function of irradiation temperature after J. H. Kittel and S. H. Paine (*7*).

authors have advanced a few predictions. One of these, namely, that the irradiation growth at very low temperatures where diffusion is very slow should be low has since been confirmed by Kunz and Holden (*2*). Another prediction concerning the dependence of irradiation growth upon the crystal size of the samples, due to experimental difficulties, has been only partly confirmed by the work of Resnick and Seigle (*3*). The prediction, however,