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LETTER TO THE EDITORS

# Reactor Transfer Functions: Addendum

In a recent issue of this journal, an application of the theory of stochastic processes to reactor kinetics was presented (*1*). The following note is intended to comment on the nature of the stochastic force. Equation numbers, notation, *et al.* refer to the aforementioned paper.

To obtain the relationship (19) which permits experimental determination of the parameters in (11) and hence the transfer function of the reactor, it was necessary to introduce a stochastic "driving force" $f(t)$ to which the system responded and produced the noise $y(t)$, cf. Eq. (13). This force had the properties (a) $\bar{f}(t) = 0$ and (b) $\rho_f(\lambda) \backsim \overline{f(t)f(t+\lambda)} = 0$ for $\lambda > \lambda_0$, $\lambda_0 \ll \tau$. That is to say, the correlation time for the input is much less than that of the response. The validity of the assumptions (a) and (b) is assured by an analysis of the physical interpretation of $f(t)$.

Since a reactor is a multiloop system, the term "reactor transfer function" is somewhat ambiguous because in fact the system is described by a transfer matrix, the elements of which represent transfer functions for the various loops. In practice the term "reactor transfer function" has been used to refer to that element of the transfer matrix which connects reactivity with power level (total fission rate) because for experimental purposes the reactivity is the parameter which is conveniently varied. However, since the reactor is defined by a coupled system of linear differential equations, it is possible to eliminate between them and obtain a single equation of order higher than any of the rest and it is that equation which appears in (11). The reason why this particular representation of the reactor is desirable for present purposes will be apparent later. Hence in this communication the term "reactor transfer function" refers to what might be called "the net transfer function." The net differential operator $Z(D)$ of Eq. (11) is in general a polynomial in $D$ of the form

$$Z(D) = \alpha_0 + \alpha_1 D + \alpha_2 D^2 + \cdots + \alpha_n D^n$$

where the parameters $\alpha_i$ contain various combinations of the parameters which appear in the original system of coupled equations. If there were no statistical processes in the reactor, the fundamental parameters, i.e., delayed neutron lifetimes, fuel density, number of neutrons per fission, void volumes, etc., would be independent of time. Due to these fundamental statistical processes, these fundamental parameters have values which fluctuate about some mean value. Since the parameters $\alpha_i$ are composed of these fundamental parameters in some fashion which can be specified from theory, the $\alpha_i$ also will be time-dependent quantities fluctuating about some mean value thus:

$$\alpha_i = \alpha_i^0 + \alpha_i'(t)$$

where

$$\bar{\alpha}_i'(t) = 0.$$

Thus (11) because of its linearity may be written

$$Z^0(D)r(t) = -Z'(D)r(t)$$

where

$$\begin{aligned} Z^0(D) &= \alpha_0^0 + \alpha_1^0 D + \alpha_2^0 D^2 + \cdots + \alpha_n^0 D^n \\ Z'(D) &= \alpha_0' + \alpha_1' D + \alpha_2' D^2 + \cdots + \alpha_n' D^n. \end{aligned}$$

Now no matter how determined $r(t)$ is, (it has at least $n - 1$ derivatives), the product $\alpha_i'(t)D^i r(t)$ is still random and $Z'(D)r(t)$ may be replaced by a random function $f(t)$ whence

$$Z^0(D)r(t) = f(t).$$

Recognizing that $r(t)$ is noise and $Z^0(D)$ is what is meant by $Z(D)$ in practice, we have immediately (13).

It is clear from the foregoing that an ensemble average of $f(t)$ vanishes and hence $\bar{f}(t) = 0$ thus justifying assumption (a) above. Furthermore no matter how random $f(t)$ , $y(t)$ must be less random since it has been produced from the network and has been smoothed. Since $y(t)$ is less random than $f(t)$, $y$ is correlated over longer times than $f$ which is just assumption (b).

Because, in principle, all the fundamental parameters could contribute fluctuations to $f(t)$ it has been convenient to represent the network of reactor equations by the net equation (11).

## REFERENCE

1. M. N. Moore, *Nuclear Sci. and Eng.* **3**, 387 (1958).

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