Pipe Thicknesses Which Minimize Stresses Due to Internal Pressure and Gamma Heating

This note presents the numerical values of pipe thicknesses, Table I, which minimize the total stress in a tube which is internally cooled and under internal pressure p with uniform gamma heating q. Such values are needed to determine the pipe thicknesses in reactors with high gamma fluxes. Total stress values as well as the contributions due to the pressure and temperature gradient are given in Fig. 1 and Table I. Table II lists the material properties of various piping materials which are useful for these calculations.

Nomenclature

p = pressure
q = internal heat generation per unit volume
S = Ratio of total circumferential stress to pressure
S_T = Ratio of circumferential stress (due to temperature) drop to pressure
S_p = Ratio of circumferential stress (due to pressure) to pressure
a = inner radius
x = (outer radius)^2/(inner radius)^2
α = linear coefficient of thermal expansion
K = thermal conductivity
E = elastic modulus
v = Poisson's ratio
B = $E\alpha/K(1 - v)$ = material property
m^3 = $12\ p/qa^2B$ = solution parameter.

General Theory

For an elastic hollow cylinder with uniform heat generation q and cooled internally, the maximum circumferential tensile stress occurs at the inner surface and is given by the following equation:

$$S_T = \frac{3}{2m^3}\left(\frac{2x^2 \ln x}{x - 1} - 3x + 1\right).$$

The circumferential stress at this same surface due to internal pressure p is

$$S_p = (x + 1)/(x - 1).$$

The total circumferential stress S is the sum of these stresses,

$$S = S_T + S_p\ .$$

The stress S can be minimized by setting

$$\frac{\partial S}{\partial x} = f(x) = 2x^2 \ln x - 4x \ln x - x^2 + 4x - \frac{4}{3} m^3 = 0$$

TABLE I

PARAMETERS FOR PIPE THICKNESS, STRESS, AND TEMPERATURE DROPS FOR PIPES WITH INTERNAL COOLING, PRESSURE, AND UNIFORM HEATING[a]

$m = \left(\frac{12p}{qa^2B}\right)^{1/3}$	$x - 1$	S_T	S_p	S	$K\Delta T/q$
0.010	0.009593	78.45	209.5	287.9	1.146×10^{-5}
0.020	0.01996	47.98	101.2	149.2	4.943×10^{-5}
0.030	0.03007	32.91	67.52	100.4	1.119×10^{-4}
0.040	0.04018	24.95	50.78	75.73	1.991×10^{-4}
0.050	0.05030	19.98	40.76	60.74	3.111×10^{-4}
0.060	0.06044	16.65	34.09	50.74	4.476×10^{-4}
0.070	0.07060	14.27	29.33	43.60	6.088×10^{-4}
0.080	0.08078	12.49	25.76	38.25	7.946×10^{-4}
0.090	0.09100	11.11	22.98	34.09	1.005×10^{-3}
0.100	0.1012	9.997	20.76	30.75	1.240×10^{-3}
0.110	0.1115	9.087	18.94	28.03	1.499×10^{-3}
0.120	0.1218	8.329	17.43	25.75	1.782×10^{-3}
0.130	0.1321	7.689	16.14	23.83	2.090×10^{-3}
0.150	0.1527	6.664	14.09	20.76	2.778×10^{-3}
0.170	0.1735	5.879	12.53	18.41	3.563×10^{-3}
0.200	0.2048	4.997	10.76	15.76	4.920×10^{-3}
0.250	0.2575	3.996	8.766	12.76	7.658×10^{-3}
0.300	0.3108	3.329	7.436	10.76	1.099×10^{-2}
0.350	0.3645	2.852	6.486	9.338	1.490×10^{-2}
0.400	0.4189	2.494	5.775	8.269	1.938×10^{-2}
0.500	0.5291	1.993	4.780	6.773	3.007×10^{-2}
0.600	0.6414	1.659	4.118	5.777	4.299×10^{-2}
0.700	0.7556	1.420	3.646	5.067	5.812×10^{-2}
0.800	0.8718	1.241	3.294	4.535	7.541×10^{-2}
1.000	1.110	0.9891	2.802	3.791	1.163×10^{-1}

[a] See nomenclature for definition of the symbols.

and solving for x. A direct solution for x can not be found, thus, Newton's method of approximation

$$x = [x_0 f'(x_0) - f(x_0)]/f'(x_0)$$

is used to set up the following iteration equation for x:

$$x - 1 = d = \frac{2(1 + d_0^2)\ln(1 + d_0) - d_0(2 - d_0) + \frac{4}{3}m^3}{4d_0 \ln(1 + d_0)}$$

where d_0 is the previous estimate of d. The initial estimate of d is

$$d_0 = m.$$

The temperature drop ΔT across the tube is

$$\frac{K\Delta T}{q} = \frac{1}{4}(x \ln x - x + 1).$$

TABLE II

PROPERTIES OF PIPING MATERIALS AT 500–600° F

Material	E	v	α	K	YS	US	B	ρ
	psi/10^6		10^{-6}	W/in°F	psi/1000	psi/1000	$E/K(l = v)$	gm/cm^3
Aluminum base								
2S-O	8.0	0.33[a]	15.9	3.261	3	5	58.2	2.69
3S-H18	8.0	0.33[a]	13.9	2.185	10.5	17	76.2	2.73
14S-T6	8.5	0.33[a]	13.6	2.185	8	10	79.0	2.80
17S-T4	8.4	0.33[a]	13.9	1.713	7	10	79.7	2.79
24S-T4	8.5	0.33[a]	13.7	2.659	10	12	79.6	2.77
51S-T6	10.21	0.33[a]	13.9	2.422	4.5	5.5	96.8	2.69
52S-H36	8.15	0.33[a]	14.3	1.950	8	12	79.8	2.68
53S-T6	10.01	0.33[a]	13.9	2.186	3.5	6	95.2	2.69
61S-T6	8.0	0.33[a]	14.1	2.186	5	7.5	77.1	2.70
Magnesium base								
Dowmetal M	6.51	0.35[a]	14.4	1.773	7.5	13	83.1	1.76
Dowmetal FS-1	6.51	0.35[a]	14.4	1.360	5.5	15	105.9	1.78
Dowmetal J-1	6.51	0.35[a]	14.4	1.122	7	12	128.3	1.80
Dowmetal O-1	6.51	0.35[a]	14.4	1.063	7.8	13.6	135.4	1.80
Nickel base								
Inconel	31	0.29	9.0	0.254	90	120	1540	8.51
Inconel-X	31	0.29	9.0	0.295	90	150	1330	8.51
K-Monel	26	0.32	8.3	0.254	100	140	1250	8.47
Stainless steel								
304	24.5	0.30	9.9	0.301	34	65	1151	7.92
310	26.2	0.30	8.9	0.236	32	83.5	1412	7.92
316	28.5	0.28[a]	9.0	0.295	27	73.5	1208	7.98
321	25.8	0.30	9.8	0.307	26	62	1176	7.92
347	26.1	0.30	9.4	0.236	32	69	1490	7.98
410	28	0.28[a]	6.1	0.405	...	74.5	585.7	7.75
430	29	0.28[a]	5.6	0.368	34	62.5	612.8	7.70
446	29	0.28[a]	5.5	0.343	47	72.5	645.7	7.60
Titanium base								
Ti-150A[b]	161	0.33[a]	5.00	0.218	70	97	547.9	4.64
Ti-150B[b]	161	0.33[a]	5.00	0.218	115	117	547.9	4.64
Ti-175A[b]	161	0.33[a]	5.00	0.218	...	...	547.9	4.64
Zirconium base								
Zircaloy-2	11.25	0.31[a]	3.61	0.200	...	50	252.7	6.55

[a] Room temperature.
[b] Annealed.
(Compiled by A. A. Zoutte.)

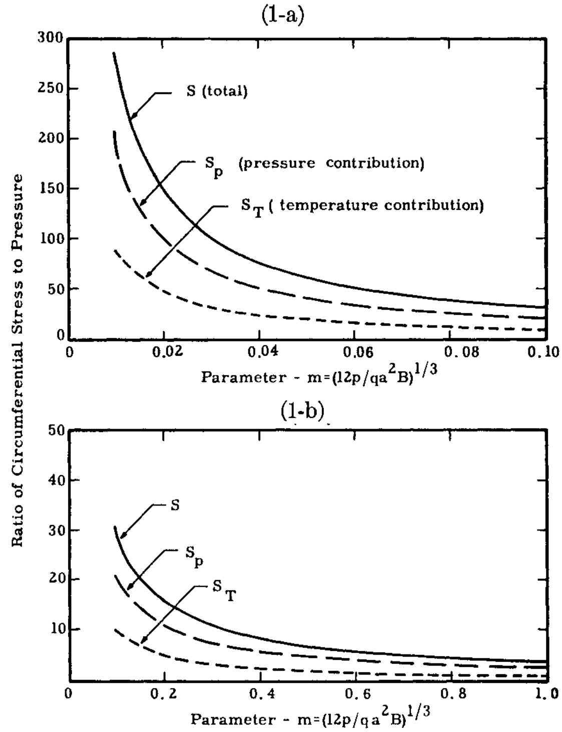

FIG. 1. Minimum pipe stresses for pipes with internal pressure and uniform heat generation.

Discussion

The numerical results presented at the end of this note can be used as an initial estimate of the minimum stresses for pressure piping in high flux reactors. Once "m" is determined as a function of material properties and operating conditions, pipe dimensions and circumferential stress are simply determined from the relation between m and x or stress given in Tables I and II.

Because of thermal gradients existing within in the pressure piping, the minimum stress condition may not be elastic. Plastic deformation reduces the stresses due to gamma heating by stress relaxation and by creating a helpful residual stress system. The plastic yielding or creep of the piping does not decrease the stress due to the pressure; thus, slightly heavier pipe than calculated by these curves should be used for those cases where maximum strength is desired. Because the combined stresses are minimized, slight alterations in pipe dimensions do not appreciably alter the total stress.

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