mizing an expression of the form

$$g_1(a_1) + \sum_{i=2}^{m} \frac{g_i(a_i)}{a_{i-1}a_i \cdots a_{m-1}} \tag{13}$$

where the sequence $\{g_i(x)\}$ is known, over the same region as above.

To treat a problem of this type introduce the sequence of functions

$$f_k(x) = \underset{\{a_i\}}{\text{Min}} \left[g_k(a_k) + \sum_{i=k+1}^{m} \frac{g_i(a_i)}{a_{i-1}a_i \cdots a_{m-1}} \right] \tag{14}$$

where the a_i are subject to

$$a_k a_{k+1} \cdots a_m = x, \qquad a_i \geq 1 \tag{15}$$

for $k = 1, 2, \cdots, m - 1$.

Then

$$f_{m-1}(x) = \underset{a_{m-1}, a_m}{\text{Min}} \left[g_{m-1}(a_{m-1}) + \frac{g_m(a_m)}{a_{m-1}} \right] \tag{16}$$

over $a_{m-1}a_m = x$, a_{m-1}, $a_m \geq 1$, and as before,

$$f_k(x) = \underset{a_k \geq 1}{\text{Min}} \left[g_k(a_k) + \frac{1}{a_k} f_{k+1}\left(\frac{x}{a_k}\right) \right]. \tag{17}$$

The computational solution is similar to that for Eq. (12).

REFERENCES

1. E. Cerrai, M. Silvestri, and S. Villani, *Z. Naturforsch.* **11a,** 694 (1956).
2. A. Mazzoni, "The Steam Vents of Tuscany and the Lardarello Plants," 2nd Edition. Calderini, Bologna, 1954.
3. R. Bellman, *Bull. Am. Math. Soc.* **60,** 503–516, (1954).
4. R. Bellman, "Dynamic Programming," Princeton Univ. Press, Princeton, to be published.
5. C. Marchetti, *Z. Naturforsch.* **9a,** 1012 (1954).

The RAND Corporation
Santa Monica, California
Richard Bellman
Received January 4, 1957

Critical Equation for the Bare Water-Moderated Reactor

A critical equation for the bare water-moderated reactor has been derived using the Goertzel-Selengut method. Neutron slowing down was assumed to be due to hydrogen alone, and the fission source was taken to be monoenergetic.

The flux at lethargy u in the slowing down region satisfies Eq. (1)

$$D(u)\nabla^2\phi(r, u) - [\Sigma_a(u) + \Sigma_{SH}(u)]\phi(r, u) + q(r, u) = 0, \tag{1}$$

with

$$D(u) = \frac{1}{3(\Sigma_{tr}(u) + \frac{2}{3}\Sigma_{SH}(u)e^{-\frac{3}{2}u})}. \tag{2}$$

The slowing down density is

$$q(r, u) = q(r, 0)e^{-u} + \int_0^u e^{-(u-u')}\,\Sigma_{SH}(u')\phi(r, u')\,du'. \tag{3}$$

Differentiating (3) with respect to u one finds,

$$\frac{\partial}{\partial u} q(r, u) + q(r, u) = \Sigma_{SH}(u)\phi(r, u). \tag{4}$$

Since the reactor is bare, the buckling at all energies is B^2. Using (1) and (4), the solution

$$q(r, u) = q(r, 0) \exp\left(-\int_0^u \frac{B^2D(u') + \Sigma_a(u')}{\Sigma_{SH}(u') + \Sigma_a(u') + B^2D(u')}\,du'\right). \tag{5}$$

On the other hand, for the neutron flux at $u = 0$ we have,

$$D(0)\nabla^2\phi(r, 0) - [\Sigma_a(0) + \Sigma_{SH}(0)]\phi(r, 0) + S(r, 0) = 0, \tag{6}$$

with the fission source $S(r, 0)$ given by

$$S(r, 0) = k_\infty(u_{th})\,\Sigma_a(u_{th})\,\phi(r, u_{th}) + \int_0^{u_{th}} k_\infty(u)\,\Sigma_a(u)\phi(r, u)\,du. \tag{7}$$

Now, $q(r, 0)$ is equal to $\Sigma_{SH}(0)\phi(r, 0)$, i.e., that portion of the neutron flux which is scattered at $u = 0$. Using (4) and (5), the integral term in (7) can be evaluated. Substituting this into (6), a solution for $\phi(r, 0)$ is obtained which is used to determine $q(r, 0)$.

If

$$\frac{B^2D(0) + \Sigma_a(0)}{\Sigma_{SH}(0) + \Sigma_a(0) + B^2D(0)} \ll 1$$

then

$$\exp\left(-\frac{B^2D(0) + \Sigma_a(0)}{\Sigma_{SH}(0) + \Sigma_a(0) + B^2D(0)}\right) \cong \frac{\Sigma_{SH}(0)}{\Sigma_{SH}(0) + \Sigma_a(0) + B^2D(0)}.$$

Substituting $q(r, u_{th})$ into the neutron balance equation (1) at thermal energy, the following critical equation is derived:

$$k_\infty(u_{th})\;\mu(u_{th})\;\frac{\exp\left[-B^2\tau(u_{th})\right]}{1 + B^2L_{th}^2} = 1 \tag{9}$$

where

$$\mu(u) = \frac{\exp\left[-\left(\frac{1}{L^2(0)} + \int_0^u \frac{du'}{L^2(u')}\right)\right]}{1 - \int_0^{u_{th}} k_\infty(u')\;\exp[-B^2\tau(u')]\;\exp\left[-\left(\frac{1}{L^2(0)} + \int_0^{u'} \frac{du''}{L^2(u'')}\right]\frac{du'}{L^2(u')}} \tag{10}$$

$$\tau(u) = \frac{D(0)}{\Sigma_{SH}(0) + \Sigma_a(0) + B^2D(0)} + \int_0^u \frac{D(u')}{\Sigma_{SH}(u') + \Sigma_a(u') + B^2D(u')}\, du' \tag{11}$$

$$\frac{1}{L^2(u)} = \frac{\Sigma_a(u)}{\Sigma_{SH}(u) + \Sigma_a(u) + B^2D(u)} \tag{12}$$

$$L_{th}^2 = D(u_{th})/\Sigma_a(u_{th}). \tag{13}$$

We wish to thank J. Chernick, T. Auerbach, and P. F. Zweifel for valuable suggestions concerning this manuscript.

REFERENCES

1. S. Glasstone and M. C. Edlund "The Elements of Nuclear Reactor Theory," Van Nostrand, New York, 1952.
2. H. Hurwitz Jr., and P. F. Zweifel, *J. Appl. Phys.* **26,** 923 (1955).

Tokyo Shibaura Electric Company
Tokyo, Japan

R. Wakabayashi
Y. Fukai

Received December 4, 1956

Engineering Test Reactors with Large Central Irradiation Cavities

The critical nuclear parameters of low-density thermally fissionable cores surrounded by moderating reflectors have been surveyed in a RAND report (*1*). The survey shows that cores with diameters in the range, ~½ to ~3 meters, require rather small fissionable masses, ~1 to ~15 kg, for criticality. Thus, the average core density is small enough to suggest the label "cavity reactor" for this unusual configuration of fissionable and moderating materials.

It was suggested that a first natural application of the cavity concept would be for test reactor purposes since a large irradiation volume is inherent to the concept. With a low critical mass, the necessary high flux is obtained if reasonable power densities may be achieved. Calculations indicate that a cavity test reactor with some 4000 liters of central irradiation space may achieve MTR flux levels while operating with MTR fuel plates under established conditions. Only computed results of pertinent characteristics are given here. Some discussion of theory may be found in reference (*1*) and a more extensive paper on both theory and applications will be published in the near future.

The general arrangement of a typical cavity test reactor is shown in Fig. 1a. This example assumes a nominal 2-meter diameter test cavity, U^{235} fuel, and heavy water for moderator and coolant. The central cavity is enveloped by an active core shell in which MTR-type U^{235}, Al fuel plates are loaded to yield the more or less standard macroscopic material densities described in reference (*2*). A critical mass of 8.5 kg of U^{235} (~90%) is required by the newly charged reactor when free of test objects. This estimate assumes that the reactor is bare beyond the reflector.

In order to estimate possible neutron flux levels, the established MTR power density, ~0.36 mw per liter of active core, may be assumed. Since the active core volume in the