Our experience has been that one iteration takes about four hours of hand computation and yields the reactivity with an accuracy of about 1%, even for rather bad flux shape guesses. This compares favorably with the same problems done by purely analytical or purely numerical methods.

For complicated internal and external reflectors, variants of the matrix method (*2*) may be used to determine the coefficients m and b in the boundary conditions.

REFERENCES

1. L. Noderer, ORNL-291, March 2, (1949), decl. January 15, (1955).
2. D. Kurath and B. I. Spinrad, ANL-4352 (1952).

Charles N. Kelber
Bernard I. Spinrad

Argonne National Laboratory
Lemont, Illinois
Received September 24, 1956

The Modal Non-escape Probability of Neutrons for Convex Bodies

Neutrons originating within or incident upon a body of arbitrary shape will suffer multiple collisions within the body before escaping or being absorbed. If the body is very small, only a few successive collisions are required for the spatial distribution to approach a limiting shape which is determined by the geometry of the body. In general, the larger the dimensions of the body in mean free paths, the greater is the number of successive collisions required for the neutron distribution to tend toward this limiting shape. In the limit of a large number of collisions, it is supposed that the nth collision generation be "multiplied" in amount sufficient to effect the same total number of collisions in the $n+1$th generation (*1*).[1] The spatial distribution in this event will be that characteristic of a "critical" assembly of the same geometry. The average nonescape probability of neutrons for this limiting distribution may then be found from the extrapolated end-point method (*2, 3, 4*) as applied to bare critical reactors.

We confine our discussion to homogeneous convex bodies which scatter neutrons isotropically without energy loss. With these conditions, the neutron flux density distribution in a bare critical assembly is obtained from the solution of the following equation

$$\varphi(\mathbf{r}) = c \int_{body} d\mathbf{r}' \Sigma \varphi(\mathbf{r}') \frac{e^{-\Sigma|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|^2} \tag{1}$$

where $\Sigma = \Sigma_s + \Sigma_a$ is the total macroscopic cross section of the reactor's constituents and c is the average number of secondary neutrons produced per collision.[2]

[1] For a fuller discussion in this limiting distribution see (*1*).

[2] As discussed in (*1*), it is usually sufficient in accounting for anisotropy of scattering to replace the scattering or elastic cross section and the total cross section by the elastic transport and total transport cross sections, respectively.

For bodies of one-dimensional symmetry, the extrapolated end-point method informs us that the asymptotic or normal mode distribution of neutrons is given by an eigenfunction $Z(\mathbf{r})$ of the wave equation, provided the associated wave number k_0 satisfies a certain characteristic equation. The appropriate wave number is found to be the positive root of the transcendental equation

$$1 = c\frac{\tan^{-1}(k_0/\Sigma)}{k_0/\Sigma} \tag{2}$$

which follows from substitution of the Fourier integral expansion of the neutron flux into both sides of the critical Eq. (1) and integration over all coordinate space.

Furthermore, if a is a typical dimension of the reactor, at the outer boundary we have the condition of the extrapolated end point

$$Z(a + z_0) = 0 \tag{3}$$

where z_0 from the end-point method is given by

$$z = (0.71044609/c\Sigma)H(c). \tag{4}$$

Boundary condition (3) implies

$$k_0(a + z_0) = \gamma \tag{5}$$

where γ is the smallest positive root of the physically pertinent eigenfunction $Z(\mathbf{r})$.

Combining (2), (4), and (5) we arrive at the following transcendental equation for $(k_0/\Sigma) = \kappa_0$

$$\frac{\tan^{-1}\kappa_0}{\kappa_0} = \frac{\gamma}{0.71H(c)\kappa_0} - \frac{\Sigma a}{0.71H(c)}. \tag{6}$$

The quantity on the left of (6) may be readily interpreted as the average nonescape probability or collision fraction F_m for neutrons in a modal distribution. In critical Eq. (1), c must be greater than unity in order to make up for the loss of neutrons by leakage from the reactor. Since transcendental Eq. (2) must be satisfied simultaneously, it follows that $F_m = \tan^{-1}(k_0/\Sigma)/(k_0/\Sigma)$ is the average nonescape probability for neutrons in a modal distribution. Solving Eq. (6) for a given Σa then gives from either side the desired collision fraction.

In (4) the quantity $H(c)$ increases monotonically from unity for $c = 1$ to 1.0556 as $c \to \infty$ (*4*). For a slab of half a mean free path in thickness the error in F_m from taking $H(c)$ to be unity is less than 1%, which is less than the error in assuming the applicability of the one-boundary extrapolated end-point results to a slab of such small dimensions. The latter error may be estimated by comparison with the average non-escape probability calculated for a constant source distribution (*4*). Figure 1 shows, for $H(c)$ equal to unity, the collision fraction F_m as a function of $X = \Sigma a$ for a sphere, an infinite cylinder and an infinite slab.

In a self-sustaining homogeneous bare reactor, the neutron spatial distribution is already in a modal or near modal distribution. In the thermal group of neutrons or in a fast group of neutrons for which an appropriate "removal" cross section may be defined, the probability that a neutron entering the group be absorbed on its nth collision is given by $(\Sigma_a/\Sigma)(\Sigma_s/\Sigma)^{n-1}(F_m)^n$. The capture fraction F_{mg} for the modal group of neutrons is then the sum of the separate probabilities giving the well-known result (*6*, *7*)

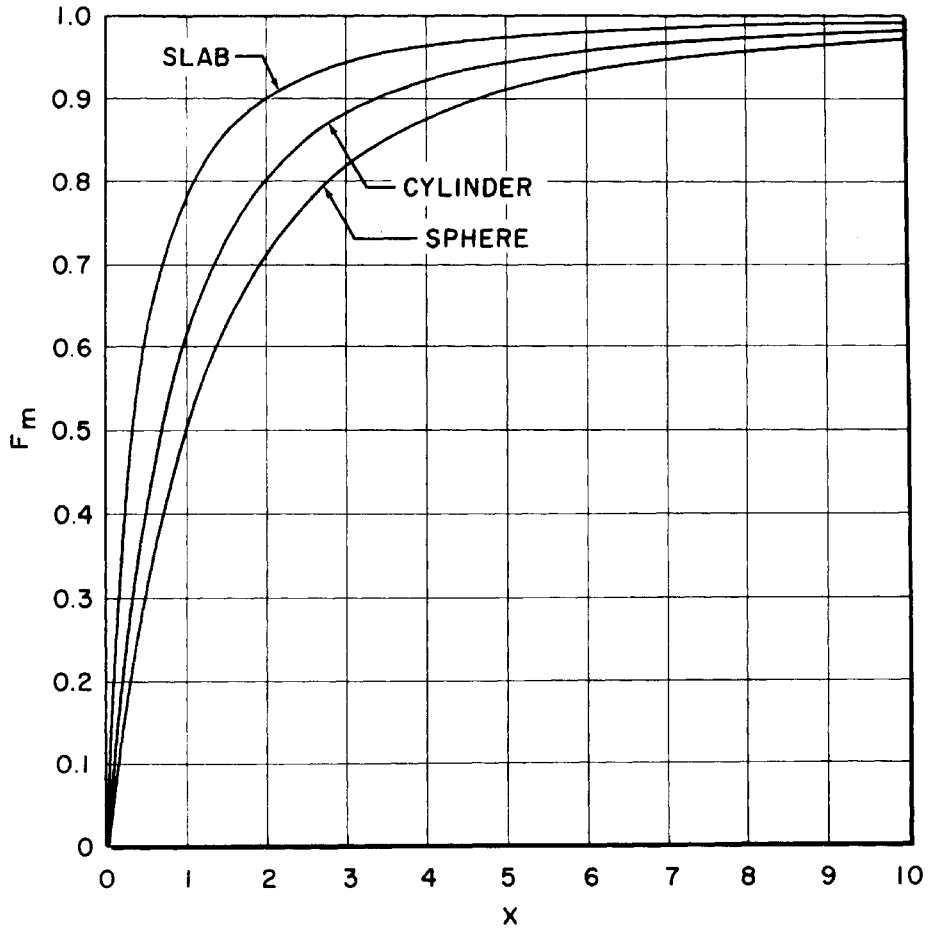

FIG. 1. Collision probability for modal distribution. For a cylinder and a sphere, X is the radius; for a slab, X is the half-thickness. X is in units of total meanfree paths.

$$F_{mg} = (\Sigma_a/\Sigma) \sum_{n=1}^{\infty} \left(\frac{\Sigma_s}{\Sigma}\right)^{n-1} (F_m)^n = \frac{(\Sigma_a/\Sigma)F_m}{1 - (\Sigma_s/\Sigma)F_m} = \frac{\Sigma_a}{[k_0/\tan^{-1}(k_0/\Sigma)] - \Sigma_s}. \tag{7}$$

For a large reactor k_0 becomes small, so that F_{mg} tends toward the elementary diffusion theory result (7):

$$F_{mg} \to [1 + (k_0^2/3\Sigma\Sigma_a)]^{-1} = (1 + L^2k_0^2)^{-1}. \tag{8}$$

In Fig. 2 are compared expressions (7) and (8) for a sphere for several absorption probabilities.

We may rewrite (6) in a form amenable to extension to other geometrical shapes, namely[3]

$$\kappa_0^2 = \gamma^2/[X + 0.71H(c)F_m(\kappa_0)]^2. \tag{9}$$

As the form of critical equation (1) is invariant under arbitrary orthogonal transformation

[3] When κ_0 is small as for a large reactor, then $H(c)$ and $F_m(\kappa_0)$ approximate unity and expression (9) becomes equivalent to the elementary diffusion theory definition for the geometric buckling (7), i.e., $B^2 = (\Sigma\kappa_0)^2$.

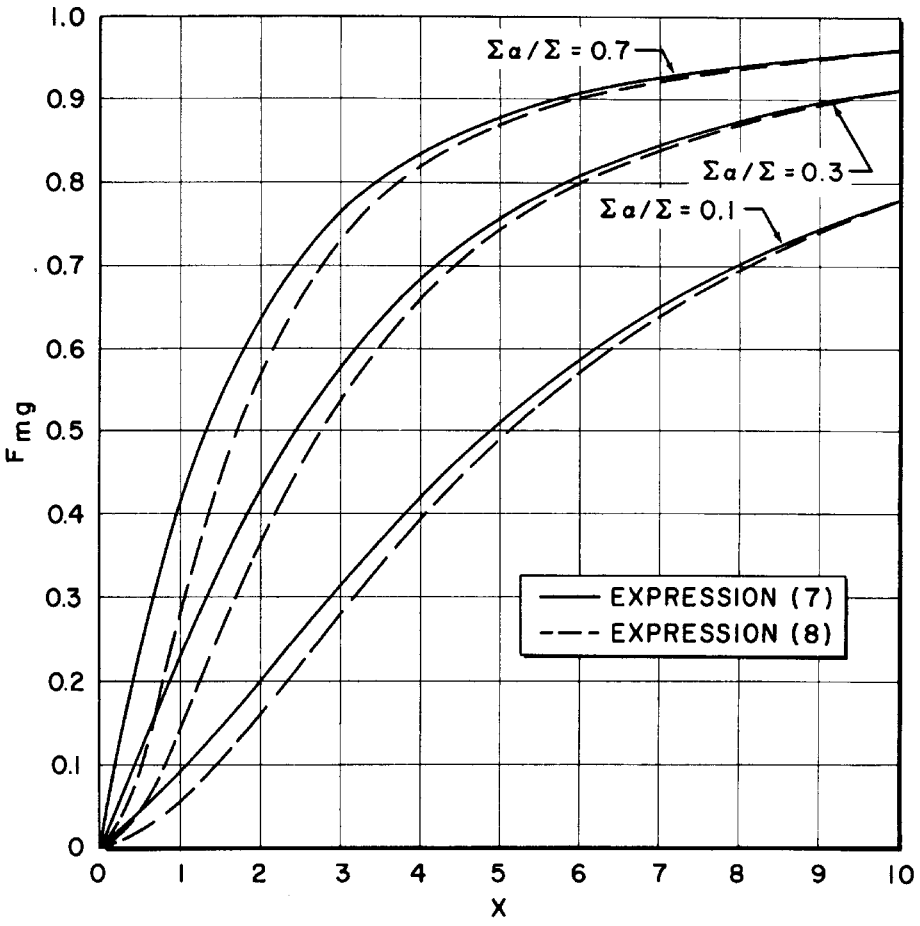

FIG. 2. Capture fraction for modal distribution for a sphere. X is the radius of the sphere in units of total mean free paths.

of coordinates, a multiple Fourier integral expansion of the neutron flux when substituted into (1) gives (2) where now

$$\kappa_0^2 = \kappa_x^2 + \kappa_y^2 + \kappa_z^2. \tag{10}$$

The nonescape probability F_m may then be found in the asymptotic sense of this note for any body for which κ_0^2 may be determined. For example, we can write immediately for a right circular cylinder

$$\kappa_0^2 = \frac{(\pi/2)^2}{(X_z + 0.71H(c)F_m(\kappa_0))^2} + \frac{(2.4048)^2}{(X_r + 0.71H(c)F_m(\kappa_0))^2} \tag{11}$$

where X_z is the half-height of the cylinder in units of total mean free paths and X_r is the radius of the cylinder in the same units. In practice the solution of (11) for the given dimensions of a cylinder is found by a rapidly converging iterative process which simultaneously determines the collision fraction F_m .

REFERENCES

1. H. A. BETHE, J. R. BEYSTER, and R. E. CARTER, *J. Nuclear Energy* **3**, 273 (1956).
2. S. FRANKEL and S. GOLDBERG, AECD-2056 (1945).

3. S. FRANKEL and E. NELSON, AECD-3497 (1953).
4. K. M. CASE, F. DE HOFFMANN, and G. PLACZEK, "Introduction to the Theory of Neutron Diffusion," Vol. I. U. S. Government Printing Office, Washington, D. C. (1953).
5. D. OKRENT, ANL-5321 (1954).
6. G. W. ANTHONY, HW-38387 (1955).
7. S. GLASSTONE and M. C. EDLUND, "The Elements of Nuclear Reactor Theory," Van Nostrand, New York (1952).

Hanford Atomic Products Operation[4] G. W. ANTHONY
Richland, Washington
Received January 21, 1957

Fission Product Recoil Separation in Suspension Reactors

The interesting paper of Wolfgang (*1*) on the use of fission product recoil separation in power reactors prompts us to comment on the use of this principle in the slurry reactor at present being developed in the Netherlands.

The use of the recoil principle has been described some years ago (*2*). Our remarks will be restricted to a slurry of uranium oxide, in water of pH about 7 as measured at room temperature.

According to their chemical behavior the fission products can be divided into four groups:

1. the gases, which will be swept out by decomposition gases (in case water decomposition is low, additional sparging can be provided);
2. ions which are not adsorbed; e.g., I^-, are to be removed by treatment of the liquid from which the fuel particles are separated;
3. ions which show a normal adsorption behavior, e.g., Cs^+;
4. ions which show an adsorption behavior of the "radiocolloidal type," e.g., the lanthanides.

The latter elements only will be discussed more extensively. The adsorption of the lanthanides at room temperature begins at pH about 3 and is virtually complete from pH = 5 upward.

Sequestering agents are effective in removing adsorbed material or preventing adsorption. This is true only at low temperature however. At high temperatures the lanthanides are extremely strongly bound to such surfaces as uranium oxide and stainless steel so that rough treatments, as boiling with 6 *N* hydrochloric acid, are then necessary for their re moval. This phenomenon alone would suffice to rule out the use of sequestering agents in reactors at high temperature, decomposition in the radiation field and thermal instability being other arguments.

With respect to these difficulties we have found it useful to apply a second solid phase in the slurry, as is also suggested by Wolfgang. Active charcoal is our preferred choice here. It has been shown that about 90% of lanthanides can be removed from the slurry system with charcoal at concentrations of the latter as low as 2 g/kg uranium oxide. Patents are

[4] Operated for the U. S. Atomic Energy Commission by the General Electric Company under Contract W-31-109-52.