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LETTERS TO THE EDITORS

Two-Group Iteration Method for Annular Cylinders

In the course of work on Argonaut, a low-power research reactor, it became necessary to investigate the critical properties of a homogeneous annular region reflected externally and internally. An iterative technique was evolved to solve these problems which was characterized by ease of application and rapidity of convergence. While the idea of an iterative technique is by no means new (*1*), the formalism used here is, to our knowledge, unique and well suited to this particular problem.

Although there are several ways of determining the critical properties of a homogeneous annulus, they are each confronted with certaini dfficulties which make solution laborious. Analytical methods enable one to satisfy the boundary conditions automatically, but the considerable algebraic manipulation of Bessel functions of nearly the same argument leads to an excessive sensitivity to minor errors in setting up the work. Straightforward solution by numerical integration over the entire volume is complicated by the problem of crossing boundaries. Hence, we have developed a technique in which uses the results of analysis to find the proper boundary conditions at the edges of the annulus and then uses numerical iteration scheme in the interior.

The two-group equations are reduced to four first-order equations by the introduction of the current as an independent variable. The interior of the annulus is divided into a number of equal intervals in each of which it is assumed that the current and the flux vary linearly. Current and flux are assumed continuous at the interval end points. The equations can then be integrated over an interval to yield easily manipulated algebraic equations giving the values of the flux at the point $i + 1$ in terms of the values at the point i. These equations have the form:

$$J_{i+1} = J_i + \lambda_i(\alpha_i J_i - \beta_i \phi_i + q_i)$$

$$\phi_{i+1} = \phi_i - \eta(J_i + J_{i+1}); \qquad \lambda_i\,,\ \alpha_i\,,\ \beta_i \text{ functions of } r.$$

The relation between the flux and the current in each group at the inner and outer boundary has the form $J = m\phi + b$, where m is a function of reflector constants only, and b is for the thermal flux, such a function times the fast flux at the boundary. For fast flux in problems with nonmultiplying boundaries, b is zero.

To start an interation, an initial shape of the slow flux is assumed and an initial value of the fast flux is guessed, the analytical boundary conditions furnishing the correct corresponding value of the current. The iteration is carried out to the outer boundary. Enough homogeneous solution (i.e., the solutions of the equations with $q \equiv 0$ and $b = 0$ at the internal boundary) is added to satisfy the analytical boundary condition there. The flux thus obtained furnishes a source term for the slow flux. A similar process yields a new estimate of the slow flux. A comparison of the weighted averages of the initial and final slow flux distributions gives an estimate of the reactivity of the system.

[A complete exposition of this method is being issued as a report of Argonne National Laboratory, ANL-5687 by B. I. Spinrad and C. N. Kelber.]

Our experience has been that one iteration takes about four hours of hand computation and yields the reactivity with an accuracy of about 1%, even for rather bad flux shape guesses. This compares favorably with the same problems done by purely analytical or purely numerical methods.

For complicated internal and external reflectors, variants of the matrix method (*2*) may be used to determine the coefficients m and b in the boundary conditions.

REFERENCES

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The Modal Non-escape Probability of Neutrons for Convex Bodies

Neutrons originating within or incident upon a body of arbitrary shape will suffer multiple collisions within the body before escaping or being absorbed. If the body is very small, only a few successive collisions are required for the spatial distribution to approach a limiting shape which is determined by the geometry of the body. In general, the larger the dimensions of the body in mean free paths, the greater is the number of successive collisions required for the neutron distribution to tend toward this limiting shape. In the limit of a large number of collisions, it is supposed that the nth collision generation be "multiplied" in amount sufficient to effect the same total number of collisions in the $n+1$th generation (*1*).[1] The spatial distribution in this event will be that characteristic of a "critical" assembly of the same geometry. The average nonescape probability of neutrons for this limiting distribution may then be found from the extrapolated end-point method (*2, 3, 4*) as applied to bare critical reactors.

We confine our discussion to homogeneous convex bodies which scatter neutrons isotropically without energy loss. With these conditions, the neutron flux density distribution in a bare critical assembly is obtained from the solution of the following equation

$$\varphi(\mathbf{r}) = c \int_{body} d\mathbf{r}' \Sigma \varphi(\mathbf{r}') \frac{e^{-\Sigma|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|^2} \tag{1}$$

where $\Sigma = \Sigma_s + \Sigma_a$ is the total macroscopic cross section of the reactor's constituents and c is the average number of secondary neutrons produced per collision.[2]

[1] For a fuller discussion in this limiting distribution see (*1*).

[2] As discussed in (*1*), it is usually sufficient in accounting for anisotropy of scattering to replace the scattering or elastic cross section and the total cross section by the elastic transport and total transport cross sections, respectively.