Letter to the Editor

Neutron-Proton Elastic Scattering

In the calculation of the transport mean free path and diffusion coefficient in a moderator, the average cosine of the angle of elastic scattering of a neutron with stationary nuclei in the laboratory system is given in most reactor physics books[1-12] as

$$\bar{\mu}_0 = 2/3A \ , \tag{1}$$

where A is the ratio of target and projectile masses. For hydrogen, A is set equal to 1. Rigorously, Eq. (1) should not be used for that case because the correct formula for scattering of a heavier particle by a lighter particle is

$$\bar{\mu}_0 = 1 - A^2/3 \ . \tag{2}$$

For the neutron of mass 1.008665 amu and the proton of mass 1.007276 amu, the value of A is 0.9986229, and thus, from Eq. (2), $\bar{\mu}_0 = 0.6675841$, which is slightly larger than 2/3. Of course, for a proton striking a stationary neutron, the usual formula Eq. (1) would apply, with $\bar{\mu}_0 = 0.6657486$.

In the general derivation with any mass ratio,

$$\mu_0 = \cos\theta \quad \text{(laboratory)} \ ,$$

$$\mu = \cos\phi \quad \text{[center of mass (c.m.)]} \ ,$$

and

$$\mu_0 = (1 + A\mu)/(1 + 2A\mu + A^2)^{1/2} \ . \tag{3}$$

A plot of Eq. (3) appears in Fig. 1 for several values of A. Note that $\mu_0(-1)$ suddenly goes from -1 for $A > 1$ to 0 at $A = 1$ to 1 for $A < 1$, corresponding to the fact that in a head-on collision, the projectile goes backward, stops, or goes forward, respectively.

The average cosine is

$$\bar{\mu}_0 = \left(\frac{1}{2}\right)\int_{-1}^{+1} \mu_0(\mu)\, d\mu \ , \tag{4}$$

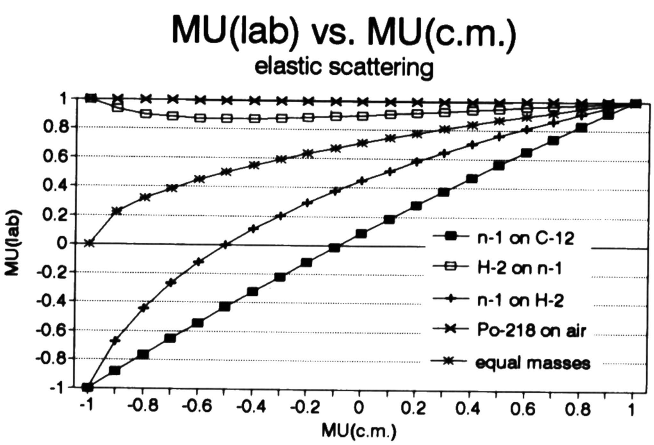

Fig. 1. Relationship of cosines of scattering angles in c.m. and laboratory systems for various mass ratios.

leading to expressions of the form $(1 - 2A + A^2)^{1/2}$. These are read as $A - 1$ if $A > 1$ but $1 - A$ if $A < 1$; hence, the final formulas are different for the two cases.

For the average logarithmic energy change ξ, the standard expression applies because α depends on the square of $1 - A$. For the neutron-proton collision, $\alpha = 4.747 \times 10^{-7}$ so that the final minimum energy is not exactly zero and $\xi = 0.9999931$.

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REFERENCES

1. S. GLASSTONE and M. C. EDLUND, *The Elements of Nuclear Reactor Theory*, D. Van Nostrand, New York (1952).

2. R. L. MURRAY, *Nuclear Reactor Physics*, Prentice-Hall, Englewood Cliffs, New Jersey (1957).

3. A. M. WEINBERG and E. P. WIGNER, *The Physical Theory of Neutron Chain Reactors*, University of Chicago Press, Chicago, Illinois (1958).

4. A. D. GALANIN, *Thermal Reactor Theory*, Pergamon Press, New York (1960).

5. R. V. MEGHREBLIAN and D. K. HOLMES, *Reactor Analysis*, McGraw-Hill Book Company, New York (1960).

6. H. S. ISBIN, *Introductory Reactor Theory*, Van Nostrand Reinhold Company, New York (1963).

7. J. R. LAMARSH, *Introduction to Nuclear Reactor Theory*, Addison-Wesley, Reading, Massachusetts (1966).

8. P. F. ZWEIFEL, *Reactor Physics*, McGraw-Hill Book Company, New York (1973).

9. A. F. HENRY, *Nuclear-Reactor Analysis*, MIT Press, Cambridge, Massachusetts (1975).

10. J. J. DUDERSTADT and L. J. HAMILTON, *Nuclear Reactor Analysis*, John Wiley & Sons, New York (1976).

11. R. A. RYDIN, *Nuclear Reactor Theory and Design*, University Publications, Blacksburg, Virginia (1977).

12. K. O. OTT and W. A. BEZELLA, *Introductory Nuclear Reactor Statics*, American Nuclear Society, La Grange Park, Illinois (1983).