Letters to the Editor

On "Simultaneous Estimation of Neutron Density and Reactivity in a Nuclear Reactor Using a Bank of Kalman Filters"

In a recent technical note, C. E. D'Attellis and E. Cortina proposed a method to estimate reactivity as well as neutron density via a bank of Kalman filters.[1] The procedure is based on Magill's method of calculating unknown parameters of stochastic processes. The adaptation of Magill's method to the case of reactivity estimation is a very attractive idea as the whole procedure seems to be quite easy and straightforward at first sight. However, every one of our attempts to reconstruct D'Attellis and Cortina's results has failed in spite of the persuasive results of the authors. Therefore, in this letter, the whole theory is carefully reviewed, and hidden difficulties are revealed. In addition, a possible explanation of why D'Attellis and Cortina's idea seemed to work is also presented.

The work in Ref. 1 is based on the following assumptions:

1. The dynamic model contains the point kinetic equations, *without* a dynamic noise term:

$$\dot{n}(t) = \frac{\rho - \beta}{l} n(t) + \sum_{i=1}^{6} \lambda_i c_i \ , \quad \beta = \sum_{i=1}^{6} \beta_i \ , \tag{1}$$

and

$$\dot{c}_i(t) = \frac{\beta_i}{l} n(t) - \lambda_i c_i \ , \qquad i = 1,\ldots,6 \ , \tag{2}$$

where

$n(t)$ = neutron density

$c(t)$ = precursor concentration

ρ = reactivity

β_i = delayed neutron fraction

l = neutron generation time

λ_i = decay constant of the precursors.

2. The scalar measurement model is the noise-corrupted observation of the neutron density:

$$z(t) = n(t) + v(t) \ , \tag{3}$$

where

$z(t)$ = signal

$v(t)$ = measurement noise.

Afterward, the continuous-time model is discretized.

3. A discrete linear Kalman filter is applied for estimating the state vector.

4. A bank of Kalman filters is used to determine the best approximation of the reactivity applying the Magill-Bogler decision rule.

The authors did not give the details of the Kalman filtering technique; they just cited the literature. So do we. Although it is a known and well-established tool for state estimation problems, some remarks must be made here. In general, only minor attention is paid to the investigation of the possible *divergence* problems of the filtering. Here divergence means an *unbounded* estimation error. Normally, divergence does not occur very often, only in extreme situations. However, the divergence does play an important role in the method just treated. The main problems are the following. In general, neither the system models nor the initial data $\hat{x}_0$ and $P_{0,0}$ are known exactly. Therefore, the model used in constructing the filter can differ from the *physical system* that generates the measurable signals. However, an inaccurate filter model *does* degrade the filter performance and could make the filter diverge. Inaccurate initial data $\hat{x}_0$ and $P_{0,0}$ can lead to similar effects as well. Fortunately, there are *theorems* that guarantee that a *wide class* of systems can be estimated *reliably* by Kalman filters without a real risk of divergence.[2,3] However, the Kalman gain K_k has the following property[2,3]:

$$Q_k \to 0 \Rightarrow K_k \to 0 \ ,$$

where Q_k is the variance of the dynamic noise. As a consequence, the lack of dynamic noise makes the filter *unable* to compensate for the fatal effect of inaccurate parameters and initial data. *Unless* every element of the system model as well as the initial data is *precise*, the estimate of the state vector *must diverge*. Schlee, Standish, and Toda[4] presented a simple analytic example to show how the estimation error becomes unbounded if a nonzero-valued control term is *ignored* in the noise-free dynamic model. At the same time, it was proven[4] that the estimation errors can be made bounded by *adding noises* to the dynamic model. Although introduction of increased noise can *improve* the stability of the estimation procedure, it could *degrade* the performance of the filter. Therefore, the optimal choice of the value of Q_k usually means certain compromise. Detailed analysis of the divergence problems can be found in Ref. 2.

Now let us consider Eq. (1) again. It is the noise-free dynamic model describing the neutron economy. According to the properties listed earlier, the Kalman filter applied to this system must be very sensitive to the parameters ρ, β_i, l, and Λ_i. Unless they are exactly known and a *precise guess* about the initial data $\hat{n}(0)$ and P_0 can be found, the estimation procedure could become *divergent*. The values of β_i, l, and Λ_i can be known reliably. However, the reactivity ρ is to be determined. In order to do it, a set of trial values of reactivity $\{\rho^i\}_{i=1}^{N}$ is chosen. Afterward, every element ρ^i of the set is substituted into the dynamic model, Eq. (1), and a Kalman filter is formed using this particular model (Magill-Bogler method[5,6]). Therefore, in each filter, the applied reactivity ρ^i as well as the system model *must* differ from the real physical system. The main constraints of this parameter estimation technique are the following:

1. It can estimate unknown parameters present only in the *control term*.

2. No divergence is allowed during the estimation procedure.

Usually, the first item is acknowledged, and the second one is ignored. The neglect of the second term could be understood from the tacit assumption that only an estimation is used that does not diverge. Consequently, it is usually believed that no *inherent* divergence occurs. However, it is not trivial and should be checked.

Unfortunately, D'Attellis and Cortina's method *does not* comply with either of these constraints. In fact, the method is meant to estimate an unknown parameter of the *state-transition matrix* $\Phi_{k+1,k}$ and not of the control term $B_k u_k$, using the Magill-Bogler procedure. The procedure may be correct, but it should be proven. The second problem arises from the filter divergence. The supposed noise-free dynamic model [see Eqs. (1) and (2)] makes the Kalman filter very sensitive to the uncertainties in the system parameters as well as in the initial conditions. To illustrate the role of the dynamic noise, some results of different computational simulation runs are presented. To this effect, the measurement data are processed by *four* different Kalman filters. For the sake of simplicity, a one-precursor-group model is used instead of the general, six-precursor-group model. The filters are different in the following two aspects: (a) the applied reactivity in the filter equations and (b) the presence or absence of a dynamic noise. The reactivity values were set to

$$\rho = 0.0023 \quad \text{and} \quad \rho = 0.0027 \ .$$

The other parameters are as follows: $\beta = 6.4 \times 10^{-3}$, $l = 9.5 \times 10^{-4}$ s, and $\lambda = 8 \times 10^{-2}\ \text{s}^{-1}$. The sampling interval is chosen to be $dt = 0.1$ s. The results are shown in Fig. 1. It can be seen that a dynamic noise term does *stabilize* the Kalman filter.

As a conclusion, even if the Magill-Bogler procedure could have been applied without changes for estimating the parameters of the state-transition matrix, the procedure must have failed because of the filter divergence. Since the Magill-Bogler procedure uses a finite set of trial values, *some* best approximation will always be obtained. Therefore, the reliability of the approximation must be checked by other means. It can be shown[7] that the score statistic of the true control parameter has a χ^2_N distribution, where N is the sampling number. Therefore, the value of $\mathcal{L}^I(N)$ corresponding to the best approximation ρ^I is a realization of a random variable with the distribution function χ^2_N. Therefore, $\mathcal{L}^I(N)$ takes its values from the *vicinity* of N. Let us apply these results to the values of Table I in Ref. 1. The minimal value of $\mathcal{L}$ is 30.30, corresponding to the best approximation of the reactivity. However, the sampling number was $N = 50$; therefore, $\mathcal{L}(N)$ should have been around 50. Certainly, a realization of $\mathcal{L}(N)$ might take the value of 30.30, but it has a very low probability (see Ref. 7 for details). In general, a very rough trial set could also produce such an effect. However, the applied resolution was $\Delta\rho/\rho \simeq 0.06$, which was dense enough. Therefore, the contradiction must be the result of the filter divergence, probably caused by omitting the dynamic noise.

Now we are in a position to give some explanation of why D'Attellis and Cortina's results were seemingly so excellent.

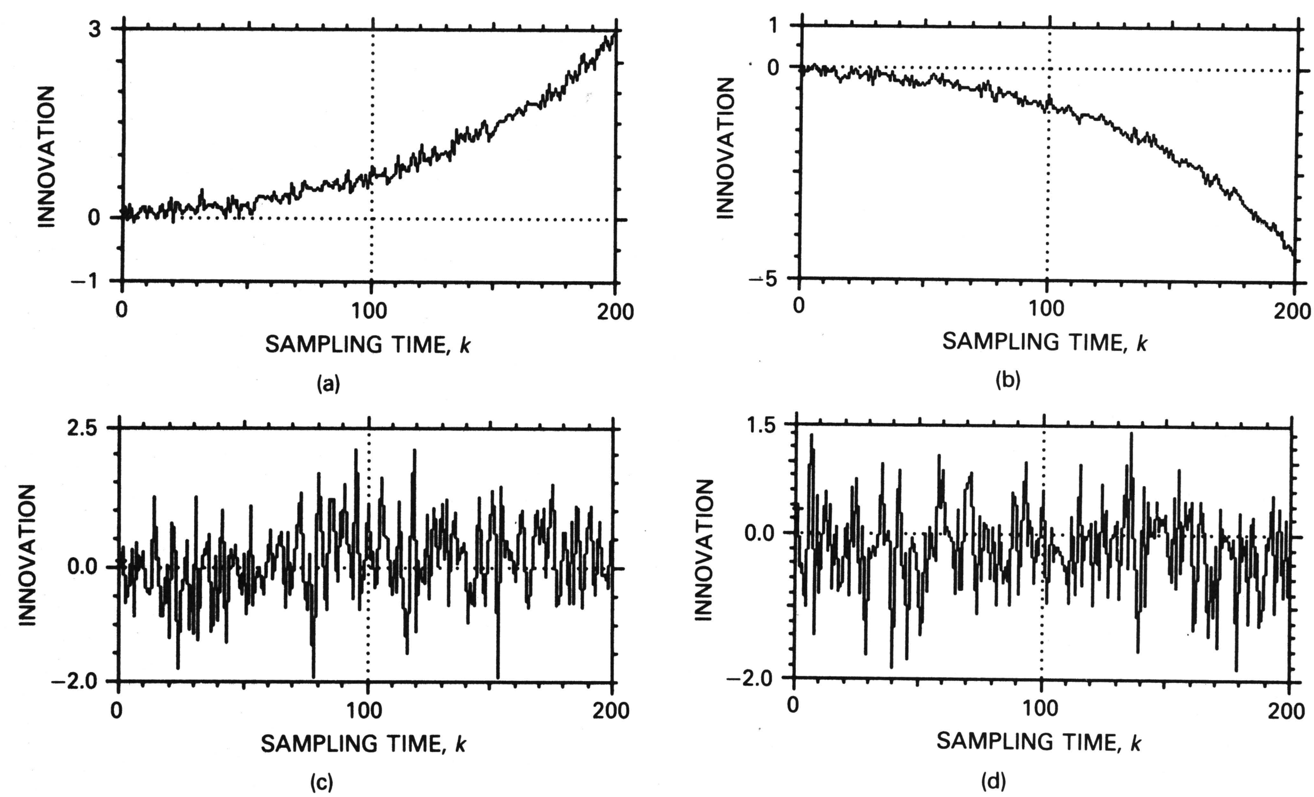

Fig. 1. The innovations: (a) $\rho = 0.0023$ and (b) $\rho = 0.0027$ *without* dynamic noise and (c) $\rho = 0.0023$ and (d) $\rho = 0.0027$ *with* dynamic noise.

Since the authors did not give any technical details about their calculation, our explanation is not more than a guess; we may be totally wrong.

There are two questions to answer:

1. How could the estimated reactivity be so close to the true value?

2. Why does the minimal value of the score statistics differ seriously from the theoretically prescribed one?

Guess 1: The processed time series was very short ($N = 50$); thus, no serious divergence was able to develop during this time (see Fig. 1). In addition, the score statistic is constructed using the *square* of the innovation; thus, *every* divergence does *increase* the score statistic. Therefore, the smaller the divergence is, the smaller the score statistic is. The amount of the divergence depends on the goodness of the approximation. As a consequence, the better the approximation is, the smaller the score statistic is.

Guess 2: It has been mentioned that $\mathcal{L}(N)$ could take such a value, which is far from its expected value, but this event has a very low probability. Therefore, in such a case, the whole calculation should be repeated again to check the reliability of the estimation. Reference 1 has not mentioned it. Finally, if the authors applied a noise-corrupted dynamic model with the score statistics given by Eq. (4), the whole problem disappears. In this case, Ref. 1 has only forgotten to mention this fact.

We pointed out that the application of the Magill-Bogler procedure in Ref. 1 is *improper* in the sense that the unknown parameter appears in the state transition matrix $\mathbf{\Phi}$ instead of the control term. This invalid interpretation of the Magill-Bogler method could also be responsible for the whole anomalous effect. It would deserve a detailed analysis to see how the Magill-Bogler technique has to be modified to be able to handle unknown parameters in the state transition matrix, too.

Inspired by D'Attellis and Cortina's idea, another procedure is developed to estimate unknown reactivities.[8,9] Supposing small changes in the reactivity, the effect can be described by the appearance of an extra input term in the point kinetic equations. The unknown reactivity shift becomes an unknown control parameter suitable for the "bank of Kalman filters" procedure.

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Reply to "On 'Simultaneous Estimation of Neutron Density and Reactivity in a Nuclear Reactor Using a Bank of Kalman Filters'"

1. The efficiency of the proposed method was checked in a simulator that matches the reactor behavior. The robustness analysis was not within the scope of our technical note.

2. $Q_k \to 0$ is a sufficient condition for $K_k \to 0$, but it is not necessary. It is very easy to construct simple examples without a dynamic noise term that verifies $K_k \to 0$ for all $P(0)$.

3. The aim of our method is to obtain a good estimation in a short time interval ($N = 50$ in the example). If the estimation time were longer, the estimation itself would not be useful. Divergencies could appear only in a long-duration Kalman filter operation.

4. Magill proposed a method for estimating a stochastic process with certain unknown parameters. According to Magill's method, the most likely filter is the filter that maximizes the (maximum *a posteriori*) probability $p(\alpha | z_k, z_{k-1}, \ldots, z_0)$ conditioned to measurement data. This is equivalent to selecting from the L hypothesized filters the filter that minimizes a sum of weighted innovations. Bogler estimates the acceleration of a maneuvering target, and the acceleration is the control variable in his model. But the important fact from Magill's analysis is the possibility of making estimations based on calculations involving the innovations. This general principle can be applied to different problems, even when the unknown parameters appear in the state transition matrix. We have successfully used this method in other fields such as acoustic emission signal analysis[1] and failure detection in a heat exchanger.[2]

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