Letter to the Editor

Comments on the Treatment of Transverse Leakage in Advanced Nodal Codes for Hexagonal Nodes

Reference 1 describes Wagner's advanced nodal code for hexagonal nodes. Wagner suggests a very interesting approach to handling the singular terms in the transverse leakage of the nodal equations for hexagonal nodes. His solution is very simple: ignore the singular terms and, to compensate, artificially modify the transversely integrated flux (TI flux). The great advantage of this approach is that the nodal equations remain essentially identical to those for rectangular nodes (except for, of course, having three instead of two sets of nodal equations on a radial plane). Wagner's argument seems to be based on intuition, but his code works surprisingly well. Wagner's method raises the intriguing question if there indeed exists a definition of the TI flux, which is transversely averaged with some kind of weighting function or along a certain transverse path so that these singular terms could be accounted for in the definition of the TI flux and would not appear in the nodal equations.

I maintain that it is possible to put Wagner's formulation on rigorous mathematical grounds via the use of conformal mapping of a hexagonal node to a rectangular node. But the resulting dimensionally reduced nodal equations, satisfied rigorously by the TI fluxes, no longer stand for a homogeneous node. The node absorption and fission cross sections appearing in the nodal equations are modified by a geometry factor that reflects local area scaling due to the conformal mapping. If one introduces the assumption that this location-dependent geometry factor could be approximated by an averaged constant, Wagner's nodal equations then result. This geometry factor depends only on the node geometry, not on any physical parameter. It can be explicitly and generically computed from the conformal mapping function. A study of this geometry factor as a function of position enables one to assess the effect of Wagner's assumption. For nodal expansion methods, where node homogeneity is not a necessary condition, this geometry factor need not be assumed constant and can be represented by nodal expansion functions.

Conformal mapping preserves the angle between intersecting curves and therefore maps a set of orthogonal curvilinear coordinates to another set of such coordinates. It has been used in electrostatics and fluid mechanics to map equipotential and field curves from one geometry to another. The electrostatic and fluid mechanics equations are Laplacian-type differential equations. The Laplacian differential operator is invariant under conformal mapping. This is why conformal mapping is useful in these two fields. Interestingly, the neutron diffusion equation is also a Laplacian differential equation, and there is the analogy of neutron current being the vector gradient of neutron flux versus electric field being the vector gradient of electric potential. The orthogonality between neutron flux and current is always preserved through conformal mappings. When mapping (orthogonal) coordinates (x,y) conformally to (u,v), the neutron diffusion equation in a homogeneous node transforms from

$$[-L^2(\partial^2/\partial x^2 + \partial^2/\partial y^2) + 1]\phi(x,y) = k\phi(x,y) \tag{1}$$

to

$$[-L^2(\partial^2/\partial u^2 + \partial^2/\partial v^2) + g^2(u,v)]\phi(u,v) = g^2(u,v)k\phi(u,v) \ , \tag{2}$$

where g^2 is the aforementioned geometry factor. This geometry factor g^2 has some very interesting properties of direct relevance to our discussion. This factor is the square of the linear scale change of going from x-y to u-v and is therefore the ratio of the local area change. Furthermore, the linear scale change g is independent of directions; that is at a given location, the medium is locally scaled by the same factor along all directions. The conformal mapping function is an analytic function transforming the complex variable $z = x + iy$ to the complex variable $w = u + iv$, and the g function is the norm of the complex derivative dz/dw. Therefore, this function g depends only on the conformal mapping function, which depends only on the node geometry.

From the foregoing two equations, one can see that if one conformally transforms a node from one to another geometry, the diffusion equation remains the same except for a modification on $k - 1$ (k is the fission multiplication factor) by the geometry factor $g^2(u,v)$. Now we map a hexagonal node to a rectangular node as shown in Fig. 1. (This must be a rectangular node, not a square node. A hexagonal node cannot be conformally mapped to a square node with four coinciding vertices.) The Cartesian grids inside the rectangular node correspond to the curvilinear grids inside the hexagonal node. Equation (2) can be reduced to one dimension by performing the straightforward transverse integration in v, and the resulting one-dimensional equation in u will not contain any singularity. The TI flux corresponds to integrating the flux $\phi(x,y)$ along a curved transverse path inside the hexagonal node. Since nodal equations are to be expressed in terms of node surface currents, it is important that the surface currents of the rectangular node are physically the same as the surface currents of the hexagonal node. This is true because the corresponding grids in the two nodes all intercept the surface boundary of the nodes perpendicularly because the mapping is conformal. However, the magnitude of the surface current of the rectangular node is scaled from the corresponding one of the hexagonal node by the inverse of the value of $g(u,v)$ at the boundary. (Being the linear scale change

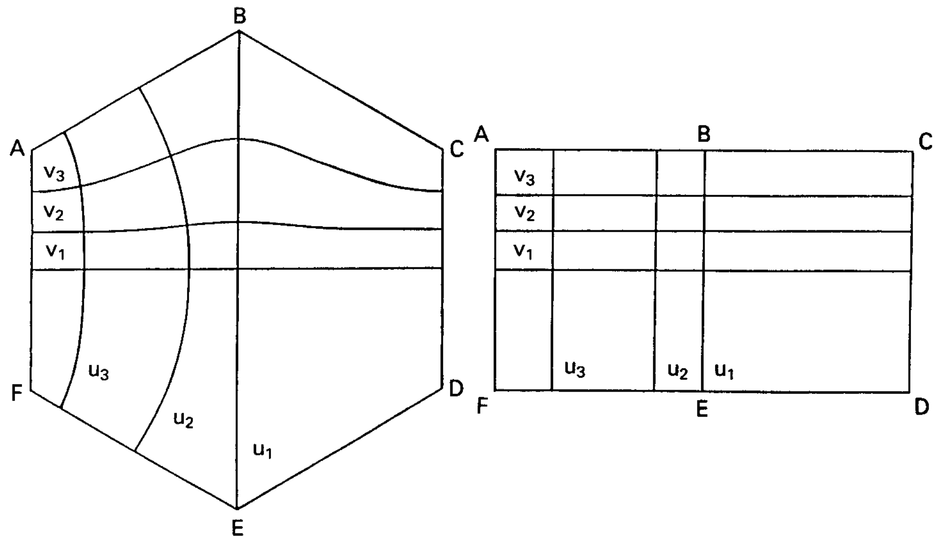

Fig. 1. Conformal mapping of a hexagon to a rectangle.

at that point, g affects the derivative there by the multiplier of $1/g$.) The same linear scale g applies to the length of the surface element since the local mapping scale is isotropic for conformal mapping as explained earlier. Therefore, not only does the surface current remain physically the same current through the mapping, but the product of the surface current and its surface element also remain invariant through the mapping. With these features established, it becomes obvious that rigorous nodal equations for hexagonal nodes that are formally identical to nodal equations for rectangular nodes can be easily derived.

Wagner's simple form of nodal equations results if one assumes that the g function in Eq. (2) can be approximated by a properly chosen "averaged" constant. Such an assumption can be quantitatively assessed by examining the behavior of the geometry factor function, which can be explicitly calculated from the mapping function. For nodal expansion methods, where the source term is to be expanded in functions, the g function need not be approximated as a constant and can be expanded as well.

Finally, the mapping function can be obtained through the use of the Schwarz-Christoffel transformation to map the hexagon to a circle and then back from the circle to the rectangle. The mapping function will result in a contour integral on a complex variable plane. The norm of the integrand of this contour integral is the scale function g, which can be numerically calculated.

After all these discussions, a note of clarification is needed. We are talking about conformally mapping a single hexagonal node to a single rectangular node. This is all that is needed to establish the desired nodal equations. This by no means implies that the whole core composed of hexagonal assemblies can be mapped to a whole core composed of rectangular assemblies. I do not think that the hexagonal assemblies can be simultaneously mapped to rectangular assemblies with a single mapping.

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REFERENCE

1. M. R. WAGNER, *Nucl. Sci. Eng.*, **103**, 377 (1989).