Letter to the Editor

Comment on the Physical Interpretation of the Flux

The usual diffusion theory derivation of Fick's law for a weakly absorbing medium[1] leads to a physical interpretation of the flux through calculation of the number of neutrons per second F, which intersect an infinitely thin foil of area A, which is placed in the $z = 0$ plane at the origin as

$$F = J_+ A + J_- A = \phi_o A/2 \ , \tag{1}$$

where J_+ and J_- are the current densities at the origin, directed upward and downward, respectively, and ϕ_o is the flux at the origin. The term J_- is calculated as a triple integral over the space above the $z = 0$ plane as

$$J_- dS = \int d\phi \int d\theta \int [r \sin(\theta)\, dr] \times \phi \Sigma_s \exp\{-\Sigma_s\}\, dS \cos(\theta)/(4\pi r^2) \ , \tag{2}$$

where dS is a differential area at the origin in the $z = 0$ plane, and Σ_s and ϕ are the macroscopic scattering cross section and the flux, respectively. To evaluate the integral, the flux $\phi(x, y, z)$ is expanded in a Taylor series about the value at the origin, and only the first spatial derivatives are retained. The value of J_+ is calculated similarly over a triple integral in the space below the $z = 0$ plane, except that the projection factor is $\cos(\pi - \theta)$. Thus, from Eq. (1), the flux may be physically interpreted as twice the rate at which neutrons strike an infinitely thin foil of unit area. This interpretation is unsatisfactory for two reasons: (a) It leaves unspecified the significance of "twice the rate," and (b) it refers to a physically unrealizable situation.

A better physical interpretation has been suggested by Hughes[2] and doubtless has been discovered by many others who have thought about the problem. The better interpretation is not only physically realizable but also can be used to indicate the reason for the words "twice the rate" in the less desirable interpretation. Consider a weakly absorbing sphere surrounding the origin whose cross-sectional area is A in the $z = 0$ plane. Then, the number of neutrons per second striking the surface of the sphere from the outside is F', which can be computed as

$$F' = J'_+ A + J'_- A = \phi_o A \ , \tag{3}$$

where J'_+ and J'_- are current densities incident on the spherical surface from the outside, from the space below the $z = 0$ plane, and above the $z = 0$ plane, respectively. The value of J'_- is calculated from a triple integral like that in Eq. (2), except that the projection factor $\cos(\theta)$ is not present, nor is $\cos(\pi - \theta)$ in the case of J'_+. Thus, from Eq. (3), the flux may be interpreted physically as the rate at which neutrons strike a weakly absorbing sphere of unit cross-sectional area.

Now, to clarify the words "twice the rate," consider a neutron that intersects the surface of such a sphere at a point b and imagine further a cross section of the sphere that contains the velocity vector of the neutron as well as the normal to the area A. In this cross section, let the intersection of area A with the left-hand surface of the sphere occur at the point a and with the right-hand surface at point c. The tangent to the cross section at point b limits the possible directions that can be taken by the neutron within the cross section to π radians. The fraction of those directions that intersect area A is defined by the angle abc. Angle abc is a right angle, since it is an angle inscribed within a semicircle. Therefore, the chance that an isotropically directed neutron that intersects the sphere also intersects the area A is 0.5. Thus, if ϕA neutrons per second intersect the sphere, then $\phi A/2$ neutrons per second will also intersect the area A. Finally, since all neutrons aiming toward area A also strike the sphere first, then $\phi A/2$ is equal to F, irrespective of whether the sphere is present or not.

REFERENCES

1. J. R. LAMARSH, *Introduction to Nuclear Reactor Theory*, p. 125 ff, Addison-Wesley Publishing Co., Reading, Massachusetts (1966).

2. D. J. HUGHES, *Pile Neutron Research*, p. 85 ff, Addison-Wesley Publishing Co., Reading, Massachusetts (1953).

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