Letters to the Editor

Comments on "The Effect of Random Material Density on Reactor Criticality"

In 1974, M. M. R. Williams[1] studied the effect of randomly dispersed fuel distribution on neutron flux and critical condition using a one-speed diffusion approximation for a bare reactor. The following equation was considered:

$$\Delta\phi(\boldsymbol{r}) + B_0^2\phi(\boldsymbol{r}) = -B_0^2\epsilon(\boldsymbol{r})\phi(\boldsymbol{r}) \ , \tag{1}$$

where

$\phi(\boldsymbol{r})$ = neutron flux

B_0^2 = critical buckling for the unperturbed reactor

$\epsilon(\boldsymbol{r})$ = random function that characterizes the small perturbation.

This equation was rewritten for the average flux $\langle\phi(\boldsymbol{r})\rangle$ and solved approximately using Green's function and asymptotic reactor theory. [Strictly speaking, if B_0^2 is the eigenvalue for the unperturbed equation, one should use the modified (generalized) Green's function,[2] but this does not change the integro-differential equation for the average neutron flux.] From this critical equation, it was concluded that "randomness increases the critical size for a given amount of fuel."[1]

If the size of a perturbed reactor is equal to the critical size of an unperturbed one, then a time-dependent equation should be used for the perturbed reactor instead of Eq. (1). The solution of this equation can be written in the following form:

$$\phi(\boldsymbol{r},t) = \phi(\boldsymbol{r})\exp\Lambda t \ ,$$

where Λ = constant is the eigenvalue of the equation for $\phi(\boldsymbol{r})$. Another possibility for reactors of the same size is to keep both of them critical, using the compensative term δ = constant as the eigenvalue for the perturbed reactor's equation:

$$\Delta\phi(\boldsymbol{r}) + B_0^2[1 + \epsilon(\boldsymbol{r}) + \delta]\phi(\boldsymbol{r}) = 0 \ . \tag{2}$$

In this equation, $\delta > 0$ can be considered as the additional multiplication, and, therefore, the reactor state that is described by Eq. (1) [or Eq. (2) with $\delta = 0$] will be subcritical. The case $\delta < 0$ may be considered as the additional absorption and, therefore, the perturbed reactor with $\delta = 0$ will be overcritical. In a time-dependent equation, it is evident that $\Lambda > 0$ corresponds to overcriticality of the perturbed reactor and $\Lambda < 0$ corresponds to subcriticality.

It was shown[3,4] that the new perturbation theory can be conveniently used for solving Eq. (2). This theory was first proposed in quantum mechanics for the Schrödinger equation and variously called the "nonlinearization method," the "logarithmic perturbation theory," and so on.[5] The advantages of this theory are that all corrections to the fundamental eigenvalue δ and the main eigenfunction $\phi(\boldsymbol{r})$ are expressed only in terms of the unperturbed eigenvalue B_0^2 and the eigenfunction $\phi_0(\boldsymbol{r})$ for which the corrections are being sought (but not in terms of the complete spectrum of eigenvalues and eigenfunctions of the problem as in the classical perturbation theory).

In one-dimensional geometry, all corrections are expressed in quadratures. Moreover, it is possible to construct such a rapid convergence for the perturbation theory series that in approximation $m \geq I$, the neutron flux and the main eigenvalue are computed to an accuracy to λ^{2^m}, where λ is the smallness parameter.[3]

It follows from the simple formulas[3,4] that in expansions

$$\Lambda = \sum_{m=1}^{\infty} \lambda^m \Lambda_m$$

and

$$\delta = \sum_{m=1}^{\infty} \lambda^{2^{m-1}} \delta_m \ , \tag{3}$$

always $\Lambda_2 > 0$ and $\delta_m < 0$ for all $m \geq 2$. Hence, if perturbation $\epsilon(\boldsymbol{r})$ is such that $\Lambda_1 = 0$ or $\delta_1 = 0$, then the perturbed reactor will be overcritical. This was first noticed by Galanin in Ref. 4. Galanin also considered the effect of random heterogeneity of the fuel distribution $\epsilon(x)$ [with the zero average value $\langle\epsilon(x)\rangle = 0$ and independent perturbations in each point of the active core of the plane reactor] on the average values of the neutron flux and the eigenvalue. For the time-dependent solution, it was shown that $\langle\Lambda_1\rangle = 0$ and $\langle\Lambda_2\rangle > 0$. It is also simple to show that for Eq. (2) in this case $\langle\delta_1\rangle = 0$ and $\langle\delta\rangle < 0$. This means that among all the possible randomly dispersed fuel distributions, in most cases one will find the overcritical distributions. In other words, the overcritical state of the perturbed reactor is the most probable for such random perturbations. However, this conclusion is in contradiction with the conclusion from Ref. 1. This contradiction will be removed if one corrects the incorrect sign in Eq. (15) of Ref. 1.

E. G. Sakhnovsky

Leningrad Nuclear Physics Institute
Gatchina, Leningrad District
188350, USSR

January 31, 1989

REFERENCES

1. M. M. R. WILLIAMS, *Atomkernenergie*, **22**, 248 (1974).

2. G. A. KORN and T. M. KORN, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill Book Company, New York (1961).

3. E. G. SAKHNOVSKY, *At. Energ.*, **42**, 141 (1977) (in Russian); see *Sov. J. At. Energy* for English translation.

4. A. D. GALANIN, *At. Energ.*, **60**, 267 (1986) (in Russian); see *Sov. J. At. Energy* for English translation.

5. A. V. TURBENER, *Uspekhi Fiz. Nauk*, **144**, 35 (1984) (in Russian); see *Sov. Phys. Usp.* for English translation.