Letters to the Editor

Time-Dependent Escape Probabilities and Chord Distribution Functions

In a recent paper, Henderson and Maynard[1] pointed out that time-dependent first-flight neutron leakage rates can be calculated from their steady-state counterparts by using Laplace transform techniques. This is possible because the transformed time-dependent Boltzmann equation is formally identical to the steady-state equation with a modified total cross section.

Henderson and Maynard did not investigate the relevance of chord theory in their paper. It is, however, easy to obtain a simple, general expression for the time-dependent first-flight escape probability in terms of the chord distribution function.

Consider a homogeneous body *B*. (For simplicity, only convex bodies will be considered.) Let $f(R)$ denote the chord distribution function of body *B*, and let $\langle R \rangle$ denote the mean chord length. Then, as originally shown by Dirac,[2] the probability that a neutron born uniformly and isotropically in body *B* will escape on its first flight is given by

$$P_e(\Sigma) = (1/\Sigma\langle R \rangle) \int_0^{\infty} [1 - \exp(-\Sigma R)] f(R)\, dR \ . \quad (1)$$

It is convenient to introduce a dimensionless length

$$x = R/\langle R \rangle \quad (2)$$

and a corresponding distribution function

$$g(x)\, dx = f(R)\, dR \ . \quad (3)$$

Then,

$$P_e(\Sigma) = (1/\Sigma\langle R \rangle) \int_0^{\infty} [1 - \exp(-\Sigma\langle R \rangle x)]\, g(x)\, dx \ . \quad (4)$$

Let $P(t)$ denote the corresponding probability that a neutron born uniformly and isotropically in body *B* at time $t = 0$ escapes on its first flight during a unit time at t. Let $\tilde{P}(s)$ denote the Laplace transform of $P(t)$. Then, as shown by Henderson and Maynard,[1] $\tilde{P}(s)$ can be obtained from $P_e(\Sigma)$ by replacing Σ with $\Sigma + s/v$, where v denotes the neutron speed. When applied to Eq. (4), this approach gives

$$\tilde{P}(s) = (v/\langle R \rangle) \times \int_0^{\infty} \left\{ \frac{1 - \exp[-(\Sigma v + s)(\langle R \rangle x/v)]}{\Sigma v + s} \right\} g(x)\, dx \ . \quad (5)$$

The inverse transform of Eq. (5) is elementary and yields

$$P(t) = (v/\langle R \rangle)\exp(-\Sigma v t) \int_{vt/\langle R \rangle}^{\infty} g(x)\, dx \ . \quad (6)$$

Thus, $P(t)$ can be obtained by integrating the dimensionless chord distribution function $g(x)$. Note that $P(t)$ factors neatly into a product. The first factor gives the initial leakage rate

$$P(0) = v/\langle R \rangle \ . \quad (7)$$

This is seen to depend only on $\langle R \rangle$ and not on the detailed shape of *B*. The second factor, $\exp(-\Sigma v t)$, is the familiar probability that the neutron avoids colliding with nuclei for at least the specified time t. The third (integral) factor, a function of $vt/\langle R \rangle$, represents the fraction of the chords in the original distribution $f(R)$ whose length exceeds vt. Clearly, shorter chords are irrelevant at time t, since neutrons exiting along them must do so prior to t.

Chord distribution functions are known for several elementary geometries.[2] For the slab and solid sphere, the corresponding integrals required in Eq. (6) are trivial and reproduce the results given in Ref. 1. For a solid infinitely long circular cylinder, the integral in Eq. (6) can also be done in closed form. The resulting expression and numerical values of $P(t)$ will be reported separately.[3]

With suitable modifications, the above theory may also be applicable to bodies containing cavities or black absorbers. In certain cases, Eq. (6) may provide a more tractable calculation of $P(t)$ than does the direct Laplace inversion of $P_e(\Sigma + s/v)$.

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2. K. M. CASE, F. deHOFFMANN, and G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, Vol. 1, Los Alamos National Laboratory (1953).

3. A. G. GIBBS, "The Time-Dependent First Flight Escape Probability for an Infinite Cylinder," submitted to *Nucl. Sci. Eng.*

Response to "Time-Dependent Escape Probabilities and Chord Distribution Functions"

In his Letter to the Editor, Gibbs[1] obtains a general expression for the time-dependent first-flight escape probability in terms of the chord distribution function. As he points out, the calculations via the chord distribution function do indeed provide an alternative method for obtaining the time-dependent escape probabilities (a method that we did not investigate) and may be a more suitable method for the simple uniform source cases considered in Ref. 2. We thank him for bringing this to our attention.

Nevertheless, there are two points in his presentation that require clarification. First, the limits of integration of the chord distribution function integral are not necessarily from 0 to ∞ but are over the minimum and maximum chord lengths in the body. Hence, in Eq. (1) of his letter, the limits of integration should be R_{min} to R_{max}, and in Eqs. (2) through (5) the limits are X_{min} to X_{max}. Second, we note that Eq. (6) does not contain any step functions even though the solutions given in Ref. 2 contain them. To clarify this, we write his Eq. (5) as

$$\tilde{P}(s) = v \cdot \int_{X_{min}}^{X_{max}} \left\{ \frac{1 - \exp[-(\Sigma v + s)(\langle R \rangle x / v)]}{\Sigma v + s} \right\} g(x)\, dx \ . \tag{1}$$

The inverse transform of the above equation is

$$P(t) = v \cdot \exp(-\Sigma v t) \times \int_{X_{min}}^{X_{max}} \left[H(t) - H\left(t - \frac{\langle R \rangle x}{v}\right)\right] g(x)\, dx \ . \tag{2}$$

Expanding the solution in Eq. (2) yields

$$P(t) = v \cdot \exp(-\Sigma v t) \left[\int_{X_{min}}^{X_{max}} g(x)\, dx\, H(t) - \int_{vt/\langle R \rangle}^{X_{max}} g(x)\, dx\, H\left(t - \frac{\langle R \rangle X_{max}}{v}\right) + \int_{vt/\langle R \rangle}^{X_{min}} g(x)\, dx\, H\left(t - \frac{\langle R \rangle X_{min}}{v}\right)\right] , \tag{3}$$

where

$$X_{min} = \frac{R_{min}}{\langle R \rangle} \quad \text{and} \quad X_{max} = \frac{R_{max}}{\langle R \rangle} \ .$$

From the expression given in Eq. (3), we do note the importance of the integration limits and their appearance in the arguments of the step function. The step functions relate the time t to characteristic lengths (chords) of the body under consideration. Using Eq. (3), the slab and solid sphere uniform source solutions in Ref. 2 are easily reproduced.

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