Letter to the Editor

Thermal-Neutron Diffusion at the Ice-Water Phase Transition

Recently Williams and Munno[1] and Williams[2] pointed out the absence of discontinuity in the diffusion coefficient, $D_0 \equiv [\overline{vD(v)}]$, of neutrons at the ice-water phase transition, independent of density change. Their measurements below room temperatures join continuously with those of Silver[3] in ice. Using a simple model which takes into account chemical binding effect, they assert that neutron scattering from water near the freezing point is the same as that from ice; i.e., water near its freezing point behaves like an infinite ice crystal. In this Letter we put forward yet another argument which supports the view that there is no discontinuity in the diffusion coefficient of neutrons at the ice-water phase transition.

Using the Boltzmann diffusion equation it can easily be shown that when $\Sigma_a(v) \ll \Sigma_s(v)$, the diffusion length, L_0, for the natural absorption cross section is given by

$$L_0^{-2} = \frac{\int v\Sigma_a(v)N(v)dv}{\int vD(v)N(v)dv} \quad , \tag{1}$$

where the various symbols have their usual meaning.

Equation (1) can be written as

$$L_0^{-2} = \frac{\int N(v)d(v)}{\int vD(v)N(v)dv} \times \frac{n\int v\sigma_a(v)N(v)dv}{\int N(v)dv}$$

or

$$D_0 = n^2 K L_0 \quad , \tag{2}$$

where

$$D_0 = \frac{\int v\,\frac{1}{[3\sigma_{tr}(v)]}\,N(v)dv}{\int N(v)dv} \tag{3}$$

and

$$K = \frac{\int v\sigma_a(v)N(v)dv}{\int N(v)dv} \quad . \tag{4}$$

In the present case, $\sigma_a(v)$ does not change at the phase transition, and if we assume that water at 0°C behaves as an infinite ice crystal so that the neutron spectrum $N(v)$ remains the same, it is clear K would be a constant. Thus we can write

$$R(D_0) = \frac{[\overline{vD(v)}]\text{ ice, }0°\text{C}}{[\overline{vD(v)}]\text{water, }0°\text{C}} = \frac{(L_0^2)\text{ ice, }0°\text{C}}{(L_0^2)\text{ water, }0°\text{C}}\left(\frac{n^2\text{ ice, }0°\text{C}}{n^2\text{ water, }0°\text{C}}\right) \quad . \tag{5}$$

Any change in the diffusion coefficient would therefore be easily reflected in the diffusion length measurements.

Using the neutron scattering kernel, proposed by Tewari and Kothari[4,5] and Gangwari et al.,[6] and other quantities, we have solved the multigroup Boltzmann diffusion equation for the diffusion length, L_0, in ice at 0°C and at five other temperatures down to -85°C. The temperature variation of the diffusion length turns out to be linear and is given by

$$L_0(t°\text{C}) = 2.87 + 0.0057t \quad , \qquad 0°\text{C} \geq t \geq -85°\text{C} \quad . \tag{6}$$

The calculated value of the diffusion length at 259°K for an ice density of 0.89 g/cm^3 is 2.87 cm which agrees very well with the measured value (2.85 ± 0.05 cm) reported by Barkov et al.[7]

The diffusion length at 0°C given by Eq. (1) is 2.87 cm, the density of ice being 0.917 g/cm^3. If the increase in the density at melting is taken into account, the value of the diffusion length would become 2.63 cm. The diffusion length of neutrons in water at 0°C reported by Fermi[8] is 2.64 cm. Thus $R(D_0)$ is very nearly equal to unity showing a continuous variation in the diffusion coefficient of neutrons at the ice-water phase transition.

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September 14, 1972

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