special case $b = 0$ must be interpreted as a casual coincidence. In fact in Ref. 5 it is shown that the Appendix given by Gyftopoulos in Ref. 2 is wrong also for $b = 0$.

It is important to note that in Ref. 3 there is provided a rigorous proof for the asymptotic stability criterion first given by Akcasu and Dalfes (Ref. 6) in 1960 and later used by Akcasu and Akhtar (Ref. 7) and Lellouche (Ref. 8) to investigate the stability of a xenon controlled point reactor with the presence of temperature feedback. In Ref. 3 it is pointed out that the criterion is correct but the proof given in Ref. 6 is incorrect, because the asymptotic stability is based on the conclusion that $dU(x)/dx \to 0$ as $x \to \infty$ if $U(x)$ is non-increasing and bounded below. This conclusion is not correct, as one can easily find counter examples. However, if $dU(x)/dx$ is uniformly continuous then $dU(x)/dx \to 0$ as $x \to \infty$ by a lemma given by Barbalat (Ref. 9). Since the uniform continuity of $dU(x)/dx$ was not proved or mentioned in Ref. 6 the criticism by Di Pasquantonio and Kappel was justified. In fact, Akcasu and Akhtar (Ref. 10) had already presented a rigorous derivation of the new criterion given by Akcasu and Dalfes following the same analytical method given in Ref. 6 and using Barbalat's lemma. Unfortunately the authors of Ref. 3 did not know of this proof, and hence, could not take it into account in their paper.

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Comments on the Neutron Transport Theory with Anisotropic Scattering

In this letter, which is concerned with one-group neutron transport with anisotropic scattering in a homogeneous medium, it will be shown that there are no discrete eigenvalues of the transformed transport equation within the region of continuous eigenvalues. The existence of discrete eigenvalues in this region has been assumed by Mika,[1] who applied the spherical harmonics method to the aforementioned problem. Zelazny[2] has already shown that in the case of one-group neutron transport with isotropic scattering in a homogeneous medium, such eigenvalues do not exist.

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Analogously to Mika,[1] the scattering function is expanded into a finite series of Legendre polynomials

$$f(\Omega' \to \Omega) = (4\pi)^{-1} \sum_{K=0}^{N} b_K p_K(\Omega' \times \Omega)$$

and introduced into Boltzmann's equation for plane geometry

$$\mu \times (\partial/\partial x)\, \Psi(x,\mu) + \Psi(x,\mu) = (c/2) \sum_{K=0}^{N} b_K p_K(\mu) \int_{-1}^{+1} p_K(\mu')\, \Psi(x,\mu')\, d\mu' \quad .$$

With help of the relation

$$\Psi(x,\mu) = \exp(-x/\nu)\, \phi(\nu,\mu) \quad ,$$

the transformed equation

$$(\nu - \mu) \times \phi(\nu,\mu) = (c\nu/2) \sum_{K=0}^{N} b_K p_K(\mu) \int_{-1}^{+1} p_K(\mu')\, \phi(\nu,\mu')\, d\mu' \tag{1}$$

is obtained. Multiplication with $p_s(\mu)$ and integration over μ from -1 to +1 yields

$$(s+1) h_{(s+1)}(\nu) - \nu[cb_s - 2(s+1)]\, h_s(\nu) + s h_{(s-1)}(\nu) = 0 \quad , \quad s = 0,1,2\ldots \tag{2}$$

where

$$h_K(\nu) = \int_{-1}^{+1} p_K(\mu)\, \phi(\nu,\mu)\, d\mu \quad , \quad K = 0,1,2\ldots \tag{3}$$

$$h_{-1}(\nu) = 0 \quad . \tag{4}$$

From Eqs. (2) to (4) $h_K(\nu) \sim h_0(\nu)$. Therefore, the normalization

$$h_0(\nu) = 1 \tag{5}$$

may be used without lose of generality. The general solution of (1) is

$$\phi(\nu,\mu) = (c/2)\,[\nu/(\nu-\mu)] \sum_{K=0}^{N} b_K p_K(\mu) h_K(\nu) + \lambda(\nu)\, \delta(\nu - \mu) \quad .$$

There are discrete eigenvalues ν_i outside the interval $[-1, +1]$ given by

$$\Omega(\nu_i) = 0 \quad \nu_i \neq [-1, +1] \quad , \tag{6}$$

where

$$\Omega(z) = 1 - cz \sum_{K=0}^{N} b_K Q_K(z) h_K(z) \quad . \tag{7}$$

Additionally, there is a continuum of eigenvalues ν in the region $[-1, +1]$ with

$$\lambda(\nu) = P\Omega(\nu) \quad \nu \epsilon [-1, +1] \quad . \tag{8}$$

The completeness of the resulting set of eigenfunctions of Eq. (1) has been shown by Mika.[1]

The nonexistence of discrete eigenvalues within the continuum of eigenvalues will be demonstrated by showing that the assumption

$$\lim_{Z \to \nu} \Omega(z) = 0 \quad \nu \epsilon [-1, +1]$$

for arbitrary N leads to a contradiction to the normalization Eq. (5). Application of Plemelj's formula to Eq. (7) yields

$$\Omega^{\pm}(\nu) = P\Omega(\nu) \pm (i\pi c/2)\ \nu \sum_{K=0}^{N} b_K p_K(\nu)\, h_K(\nu)$$

from which

$$(i\pi c/2)\ \nu \sum_{K=0}^{N} b_K\, p(\nu) h_K(\nu) = 0 \tag{9}$$

$$P\Omega(\nu) = 0 \quad . \tag{10}$$

Multiplying the well-known recurrence relation for Legendre polynomials

$$(K+1) p_{K+1}(\nu) - (2K+1) \times \nu \times p_K(\nu) + K\, p_{K-1}(\nu) = 0$$

for $K = 0, 1, 2, \ldots$.

with $h_K(\nu)$, and relation [Eq. (2)] with $p_K(\nu)$, subtracting the resulting equations and performing the summation over K from 0 to N yields

$$c\nu \sum_{K=0}^{N} b_K\, p_K(\nu)\, h_K(\nu) \equiv (N+1)\, [p_{N+1}(\nu) h_N(\nu) - p_N(\nu) h_{N+1}(\nu)] \quad . \tag{11}$$

Using the recurrence relation for the Legendre functions of the second kind

$$(K+1) Q_{K+1}(\nu) - (2K+1) \times \nu \times Q_K(\nu) + K \times Q_{K-1}(\nu) = 0$$

for $K = 1, 2, \ldots$.

and the definition [Eq. (7)] of $\Omega(z)$ gives

$$P\Omega(\nu) \equiv -(N+1)\, [PQ_{N+1}(\nu)\, h_N(\nu) - PQ_N(\nu)\, h_{N+1}(\nu)] \tag{12}$$

by induction from N to $N+1$.

Inserting Eqs. (11) and (12) into Eqs. (9) and (10), respectively, mutiplying Eq. (10) with $p_N(\nu)$ and Eq. (9) with $PQ_N(\nu)$ and subtracting the resulting equations yields

$$[p_N(\nu) PQ_{N+1}(\nu) - p_{N+1}(\nu) PQ_N(\nu)]\, h_N(\nu) = -(N+1)^{-1} h_N(\nu) = 0$$

from which

$$h_N(\nu) = 0 \text{ for arbitrary } N.$$

Similarly, starting with the multiplication of Eq. (10) with $p_{N+1}(\nu)$ and Eq. (9) with $PQ_{N+1}(\nu)$ one finds

$$h_{N+1}(\nu) = 0 \quad .$$

Thus, by repeated application of Eq. (2) $h_0(\nu) = 0$, which is in contradiction to Eq. (5).

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