NUCLEAR SCIENCE AND ENGINEERING: 23, 402-405 (1965)

Letters to the Editors

Decay Constant of a Neutron Pulse Inside a Finite Solid Moderator Assembly

Recently, extensive theoretical work[1-5] has been done on the decay of a neutron pulse inside a finite solid moderator assembly that has a direct bearing on the interpretation of results of pulsed-neutron experiments. According to Corngold and others[1-5] the discrete eigenvalues of the decay constant λ, are limited by the inequality

$$\lambda \leq (v\,\Sigma_s)_{\min} \equiv \lambda_{\lim}, \qquad (1)$$

where Σ_s is the macroscopic inelastic scattering cross section for neutrons of velocity v. The minimum value of $v\Sigma_s$ occurs for $v \to 0$, and for beryllium at room temperature it is nearly 4200 sec^{-1} (we will only consider beryllium here). As Corngold and Michael[4] point out, the experimental values of the lowest decay constant λ_0, exceed this limit in many cases. This has lead to considerable confusion regarding the validity of the experimental results. As these authors put it, "If the experimental points are correct, they stand in direct contradiction to rather direct consequences of the Boltzmann equation . . . It is perhaps more reasonable to suppose that the measurements in crystalline moderators at large B^2 have a large uncertainty attached to them . . ." (Here B^2 is the buckling for the assembly and for a cube of side L, B^2 is very nearly equal to $3\pi^2/L^2$.)

Corngold and Michael[4] have further pointed out that the asymptotic flux should peak at $v = 0$ and not at the Bragg peak as suggested by Jha[6] and de Saussure[7].

The purpose of this letter is to try to explain why the experimental values of λ for large B^2 come out to be larger than the mathematical bound $\lambda_{\lim}$.

Let us consider the Boltzmann equation in the diffusion approximation (in the present case one may omit any $1/v$ absorption term without loss of generality):

$$(-\lambda + v\Sigma_s + \tfrac{1}{3}v\lambda_{tr}B^2)\,\chi(E) = v\int \Sigma_s(E' \to E)\,\chi(E')\,dE', \qquad (2)$$

where λ_{tr} is the transport mean free path for neutrons of energy E, $\Sigma_s(E' \to E)$ is the scattering kernel for neutrons of energy E' being scattered into energy E, and $\chi(E)\,dE$ is the flux of neutrons in the energy range E and $E+dE$. The other symbols have already been explained.

The eigenfunction corresponding to the lowest discrete eigenvalue must be positive for all values of E and this leads to the inequality,

$$\lambda < (v\Sigma_s + \tfrac{1}{3}v\lambda_{tr}B^2)_{\min}. \qquad (3)$$

Since the minimum value of the expression on the right-hand side of Eq. (3) occurs for $v \to 0$ (for small v; $\Sigma_s \propto 1/v$ and $\lambda_{tr} \propto v$), we get the Corngold limit, Eq. (1), for the lowest eigenvalue of Eq. (2).

It has been surmized that a possible reason for the observed λ exceeding $\lambda_{\lim}$ is that the measurement of λ is made without waiting for 'long enough' time[4]. We suggest that *the observed values of λ are the eigenvalues of a bounded equation*, i.e. eigenvalues of Eq. (2) when it is suitably bounded on the low-energy side. This suggestion is significant as the calculated values of λ do not depend sensitively on the cutoff energy (provided it is not too close to zero). Physically it implies that during times relevant to an experimenter, the entire neutron-energy distribution, except for neutrons below the cutoff, remains in equilibrium and the distribution decays as a single exponential.

To test the above idea we have solved Eq. (2) for a cutoff in energy at 10k (k is the Boltzmann constant). Moving the cutoff down to 5 or 6k does not alter the results appreciably. We find that 1) *a discrete eigenvalue and a proper eigenfunction exists for each B^2 investigated* ($0 \leq B^2 \leq 0.07$ cm^{-2}) and 2) *the results for λ agree remarkably well with the experimental results* of Andrews[8] (Fig. 1). (There is a large uncertainty[8] in the experimental value of λ for $B^2 = 0.0746$ cm^{-2}.)

One further finds that for large B^2 the calculated flux peaks in the energy range corresponding to the largest Bragg peak (for details see Goyal and Kothari[9]). This is expected since, if we neglect the 'zero-energy neutrons,' for a crystalline moderator the function $(v\Sigma_s + \frac{1}{3}v\lambda_{tr}B^2)$ will have a minimum at the largest Bragg peak and the neutrons will peak at this minimum. Instead of the Corngold limit, the physically much-more-significant limit set on λ will be

$$\lambda < (v\Sigma_s + \tfrac{1}{3}v\lambda_{tr}B^2)_{E_0} \equiv \lambda_K, \qquad (4)$$

where E_0 is the energy corresponding to the largest value of Σ_{tr}, which in the case of beryllium is 80k. As is seen from Fig. 1, all experimental points, except for the last point which is rather uncertain[8], lie well below the line $\lambda = \lambda_K$.

It is interesting to note here that the decay constant for an assembly can also be deduced by averaging $D(E)\,B^2 \equiv \frac{1}{3}v\lambda_{tr}(E)\,B^2$ over the equilibrium flux distribution for that assembly. Having calculated the equilibrium flux by solving by numerical interation the bounded Boltzmann equation, it is simple to average $D(E)\,B^2$ over that distribution. The values of λ so deduced agree, as expected, with the calculated eigenvalues for different assemblies.

[1] N. CORNGOLD, P. MICHAEL and W. WOLLMAN, *Proc. Brookhaven Conf. Neutron Thermalization*, 1103 (1962); *Nucl. Sci. Eng.*, **15**, 13 (1963).

[2] M. NELKIN, *Physica*, **29**, 261 (1963).

[3] N. CORNGOLD, *Nucl. Sci. Eng.*, **19**, 80 (1964).

[4] N. CORNGOLD and P. MICHAEL, *Nucl. Sci. Eng.*, **19**, 91 (1964).

[5] C. S. SHAPIRO and N. CORNGOLD, *Phys. Rev.*, (to appear).

[6] S. S. JHA, *J. Nucl. Energy*, **A12**, 89 (1960).

[7] G. de SAUSSURE, *Nucl. Sci. Eng.*, **12**, 433 (1962).

[8] W. M. ANDREWS, "Measurement of the Temperature Dependence of Neutron Diffusion Properties in Beryllium Using a Pulsed Neutron Technique," UCRL 6083 (1960).

[9] I. C. GOYAL and L. S. KOTHARI, *Nucl. Sci. Eng.*, (to appear).

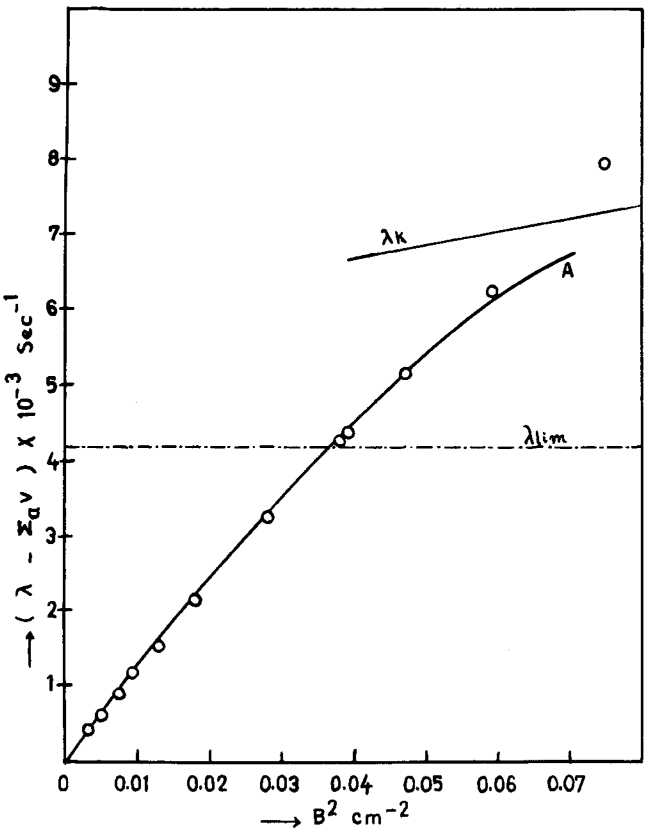

Fig. 1. Curve A shows the calculated values of $(\lambda - \Sigma_a v)$ as a function of B^2 for beryllium at room temperature. The broken line shows λ_{lim} whereas the full straight line gives λ_K. Circles are the experimental points of Andrews[8].

We would like to mention at the end that the eigenvalues of the complete Eq. (2) without the cutoff in energy will dominate only after long times but by then most of the neutrons would have leaked out.

Details of this work are to be reported shortly.

ACKNOWLEDGMENTS

I would like to acknowledge many interesting discussions with D. S. Kothari. I am also thankful to A. K. Ghatak for discussions.

L. S. Kothari

University of Delhi
Dept. of Physics and Astrophysics
Delhi, India

Received March 4, 1965
Revised April 27, 1965

Decay Constant of a Neutron Pulse

Dr. Kothari[1] has shown that if one uses a particular model for the scattering of neutrons by beryllium, and works with a cut-off scale of velocities, one will obtain the values of $\lambda_0(B^2)$ measured by Andrews. This is an interesting result. It is not an explanation, because it rests upon the ad hoc notion that one *should* limit the range of neutron velocities. We are aware of no physical principle or experimental constraint that compels one to cut-off at ten degrees, or at five degrees, especially when the experiment is marked by a strong diffusion-cooling effect. If the physicist observes an exponential decrease characterized by $\lambda > \lambda^* = (v\Sigma)_{min}$, the phenomenon must be understood through reasoning based upon the Boltzmann equation in the full domain of the velocity variable, v.

It is not at all difficult to find a qualitative explanation for this phenomenon; indeed, Dr. Michael and I convinced ourselves of one in 1962, when we first discussed the bounds on the discrete λ's. It is this: When the system is small enough, no discrete λ's will exist, and the evolution of the pulse will be described in terms of a continuous spectrum of decay constants. Then, the amplitude, $A(\lambda)$, which is associated with the λ's between λ and $\lambda + d\lambda$, will play a particularly important role. $A(\lambda)$ will reflect the scattering properties of the moderator; in the case of a coherent crystalline sample, it will show considerable oscillation, while it will vary smoothly when the moderator is an incoherent scatterer. Since a sharp peak (or valley) in $A(\lambda)$ at $\lambda = \lambda_P > \lambda^*$ produces an effect upon integration, rather like that of a discrete mode $\sim \exp(-\lambda_P t)$, one sees that such a pseudo-mode may well be found in a coherent scatterer. Further, we shall see that the value of λ_P one obtains lies close to that suggested earlier by de Saussure[4]. Of course, λ_P, while it may dominate the decay of the pulse, is in no way connected with a fundamental or asymptotic mode. After a sufficiently long time, the portion of the continuous spectrum in the neighborhood of λ^* will dominate the decay.

I can make the argument more quantitative by treating the leakage of neutrons by diffusion theory. (The diffusion approximation is hardly justified, but it yields the main features of the argument.) Then, for sufficiently large buckling,

$$N(v,t) = \int_{\lambda^*}^{\infty} d\lambda e^{-\lambda t} A(\lambda, v). \tag{1}$$

One can show, now, that[2]

$$A(\lambda,v) = \left[P \frac{1}{(v\Sigma - \lambda)} + f(\lambda)\,\delta(v\Sigma - \lambda)\right] g(\lambda,v), \tag{2}$$

where P denotes 'principal value,' f and g are 'smooth' in λ, and

$$v\Sigma \equiv v\Sigma_{s,\text{inel}} + vD(v)B^2. \tag{3}$$

The response, $R(t)$, of a $1/v$ detector to the pulse will be given by the integral of Eq. (1) with respect to v Equation (2) tells us that the result is

$$\int_0^{\infty} dv N(v,t) = \int_{\lambda^*}^{\infty} d\lambda\, e^{-\lambda t}\left[B(\lambda) + \frac{f(\lambda)g(\lambda, v(\lambda))}{\left|\frac{d}{dv} v\Sigma\right|_{v(\lambda)}}\right] \tag{4}$$

In Eq. (4) $v(\lambda)$ is the solution to $v\Sigma(v) = \lambda$, an equation assumed, for simplicity, to have only one solution. The quantity in square brackets is the amplitude, $A(\lambda)$, mentioned above. Its fluctuating nature stems from the denominator of the second term. When coherent scattering

[1]L. S. KOTHARI, *Nucl. Sci. Eng.*, this issue, p. 402.

[2]See, for example, R. BEDNARZ and J. MIKA, *J. Math. Phys.*, **4**, 1285 (1964).

[3]These conclusions are illustrated by the recent numerical calculations of A. GHATAK and H. C. HONECK, *Nucl. Sci. Eng.*, **21**, 227 (1965); *J. Nucl. Eng.*, **19**, 1 (1965).

[4]G. de SAUSSURE, *Nucl. Sci. Eng.*, **12**, 433 (1962).