In conclusion we may say:

a) Thie's MC values are good, and the disadvantage factor does not decrease monotonically with decreasing moderator density.
b) Integral transport methods are to be preferred over differential methods when calculating disadvantage factors for tightly packed lattices.
c) The WS approximation with white-boundary conditions[5,11] seems to be a good approximation for use with integral methods.

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On the Variational Method Applied to the Monoenergetic Boltzmann Equation

In two papers[1] published in this journal, Pomraning and Clark have discussed the separation of the non-self-adjointness from the monoenergetic Boltzmann equation. The authors, however, have been unable to find the proper boundary terms which, added to the functional, would yield the boundary conditions corresponding to the adjoint Boltzmann equation. It is proposed to use the alternate set of boundary conditions for which the boundary terms in the functional are relatively simple to obtain.

Let us take the Boltzmann equation in the following form:

$$\mu \frac{\partial \psi}{\partial z} + \sigma \psi(z,\mu) = c\sigma \sum_{n=0}^{\infty} \frac{2n+1}{2} f_n P_n(\mu) \int_{-1}^{+1} d\mu' P_n(\mu')\psi(z,\mu'). \tag{1}$$

The notation is identical with that used in Reference 1.

[1]G. C. POMRANING and M. CLARK, Jr., "The Variational Method Applied to the Monoenergetic Boltzmann Equation." Part I and II. *Nucl. Sci. Eng.*, **16**, 147-164 (1963).

The angular distribution $\psi(z,\mu)$ can be split into the even and odd part with respect to μ:

$$\psi(z,\mu) = \psi^+(z,\mu) + \psi^-(z,\mu). \tag{2}$$

Accordingly the even and odd parts of Equation (1) are satisfied separately:

$$\mu \frac{\partial \psi^-}{\partial z} + \sigma\psi^+(z,\mu) - c\sigma \sum_{\text{even}} (2n+1) f_n P_n(\mu) \int_0^1 d\mu' P_n(\mu')\psi^+(z,\mu') = 0 \tag{3}$$

$$\mu \frac{\partial \psi^+}{\partial z} + \sigma\psi^-(z,\mu) - c\sigma \sum_{\text{odd}} (2n+1) f_n P_n(\mu) \int_0^1 d\mu' P_n(\mu')\psi^-(z,\mu') = 0. \tag{4}$$

Both equations are valid for μ belonging to the interval (0,1). By eliminating $\psi^-(z,\mu)$ or $\psi^+(z,\mu)$ one can get an equation for $\psi^+(z,\mu)$ or $\psi^-(z,\mu)$, respectively. For example, solving Equation (4) for $\psi^-(z,\mu)$ we get:

$$\psi^-(z,\mu) = -\frac{1}{\sigma}\mu\frac{\partial \psi^+}{\partial z} - \frac{1}{\sigma}\sum_{\text{odd}} (2n+1)\frac{cf_n}{1-cf_n} P_n(\mu)\int_0^1 d\mu' P_n(\mu')\mu' \frac{\partial \psi^+(z,\mu')}{\partial z} \tag{5}$$

and then the equation for $\psi^+(z,\mu)$ is:

$$\mu^2 \frac{\partial^2 \psi^+}{\partial z^2} - \sigma^2 \psi^+(z,\mu) + c\sigma^2 \sum_{\text{even}} (2n+1) f_n P_n(\mu)\int_0^1 d\mu' P_n(\mu')\psi^+(z,\mu') + \sum_{\text{odd}} (2n+1)\frac{cf_n}{1-cf_n}\mu P_n(\mu)\int_0^1 d\mu' P_n(\mu')\mu' \frac{\partial^2 \psi^+(z,\mu)}{\partial z^2} = 0. \tag{6}$$

It is the self-adjoint form of the Boltzmann equation.

To find the proper boundary conditions for Equation (6) let us consider the albedo problem for a slab. We have

$$\psi(a,\mu) = A^+(\mu) \qquad (0 < \mu \le 1) \tag{7a}$$

$$\psi(b,\mu) = A^-(\mu) \qquad (-1 \le \mu < 0)\,. \tag{7b}$$

Taking into account Equation (5) we can easily find boundary conditions for Equation (6).

These are