mation. The IR integral for this case is 3.65 b. A numerical integration of the equations gives 3.76 b, so that in comparison, the *IR* result has a 2.9% error. On the other hand, the *NR-WR* result is 4.02 b, which has a 6.9% error.

For heavier moderators the errors are even more significant. For a carbon-uranium mixture such that σ_m = 20.2, the solution to Eqs. (9) for this resonance is (κ, λ) = (0.427, 0.0042). Here the moderator *NR* approximation is clearly no longer applicable and the *NR-WR* approximation has a 56% error, whereas the *IR* result agrees to within 1.9% of the numerical result. These results are summarized in Table I.

TABLE I

Resonance Integrals (in barns) for the 6.68 eV
Resonance of U^{238} ($\sigma_m + \sigma_p$ = 30.2)

Moderator	NR-WR	IR	Numerical
Hydrogen	4.02	3.65	3.76
Carbon	4.02	2.63	2.58

It is clear, therefore, that the *NR* approximation is not always applicable to the moderator. It is particularly bad for the lower energy resonances and the heavier moderating elements. For the higher energy resonances the error of the moderator *NR* approximation is less significant. Furthermore, as the moderator concentration increases, the error decreases because the flux depression in the resonance decreases and the resonance integral becomes less sensitive to the various approximations. In all cases, however, it is not difficult to solve the appropriate *IR* equations (9a,b) and obtain the corresponding resonance integral (5) for any resonance and any mixture.

The extension to nonhomogeneous systems may be made by representing the lump by an effective scattering cross section s. As in Ref. 5, s is given in terms of the lattice characteristics C and $\bar{\ell}$ by

$$s = \frac{1 - C}{N \bar{\ell}} . \tag{10}$$

All of the results in this paper now apply, with the only change being that $\beta_{\kappa\lambda}$ is given by

$$\beta_{\kappa\lambda}^2 = 1 + \frac{\sigma_0}{s + \kappa\sigma_m + \lambda\sigma_p} \frac{\Gamma_\gamma + \lambda\Gamma_n}{\Gamma} . \tag{11}$$

The σ_m in Eq. (11) now applies to any moderator which may be admixed with the fuel.

We have used the equivalence expressed by Eq. (10) in order to keep our results simple, but its use for nonhomogeneous systems introduces addi-

tional approximations. In particular, the moderator outside the lump is necessarily treated in an *NR* approximation. An appropriate example would be UO_2 fuel rods in water. Here, σ_m would be the scattering cross section of the oxygen, treated in an *IR* approximation, while s represents the water scattering, as treated in an *NR* approximation.

Rubin Goldstein *

University of California
Berkeley, California

*Present Address: Brookhaven National Laboratory, Upton, New York.

On Transfer Cross Sections Between Overlapping Thermal Groups

In a system of moderators at different temperatures, neutrons which have been thermalized in one medium may diffuse into another, where they may suffer collisions and eventually enter the energy-distribution characteristic of this medium. The rate of transfer of neutrons from the first distribution into the second is characterized by the so-called 'transfer' or 'rethermalization' cross sections. This process of rethermalization has been investigated by several authors[1-7] in different ways.

The present note intends to demonstrate the connection between the results of the mentioned authors, by investigating concisely the case of two purely scattering moderators at different temperatures (Kottwitz's problem).

We assume that the thermal flux Φ in the system can be approximated by a superposition of two appropriate energy distributions ϕ_i (E) $(i = 1,2)$ weighted by spatially dependent factors:

$$\Phi(\mathbf{r},E) = \sum_{i=1}^{2} F_i(\mathbf{r}) \cdot \phi_i(E) . \tag{1}$$

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It is well-known that in a general way the amplitudes $F_i(r)$ can be determined by rendering stationary a functional. In the diffusion approximation the resulting system of partial differential equations for the $F_i(r)$ is:

$$\sum_{i=1}^{2} \left\{ -\nabla D_{ji} \nabla + [\Sigma_a]_{ji} - H_{ji} \right\} F_i(r) = S_j(r) \qquad (2)$$

$$(j = 1,2)$$

where

$$A_{ij} = \int dE \; \psi_j(E) \; A(r,E) \; \phi_i(E)$$

$$S_j = \int dE \; \psi_j(E) \; S(r,E)$$

$S(r,E)$ = source distribution at a point r

$D(r,E)$ = diffusion coefficient at a point r

$\Sigma_a(r,E)$ = macroscopic absorption cross section at a point r

$H\phi(E) = \int dE' \Sigma_s(r,E'\to E)\phi(E')-\Sigma_s(r,E)\phi(E)$

$\Sigma_s(r,E'\to E)$ = angle integrated macroscopic differential cross section for the energy change $E' \to E$

$\Sigma_s(r,E) = \int dE' \Sigma_s(r,E \to E')$ = total scattering cross section.

The $\psi_j(E)$ are the adjoint energy distributions occurring in the approximation

$$\Psi(r,E) = \sum_{i=1}^{2} G_j(r) \; \psi_j(E) \qquad (3)$$

of the adjoint flux in analogy with Eq. (1).

By operating on Eq. (2) with the inverse of the matrix

$$I_{ij} = \int \psi_j(E) \; \phi_i(E) \; dE \qquad (4)$$

(which exists if all the energy components ϕ_i, ψ_j are linearly independent), it is straightforward to derive equations in the multigroup form:

$$-\nabla D(r)\nabla F_j(r) + \Sigma_{jj} F_j(r) = \Sigma_{ji} F_i(r) + S_j(r) \qquad (5)$$

$$(i \neq j ; j = 1,2)$$

($D(r,E)$ is assumed here as energy independent) and to identify the coefficients which act in the formalism as transfer cross sections:

$$\Sigma_{ji} = -\sum_{k=1}^{2} I_{jk}^{-1} [\Sigma_a - H]_{ki} , \qquad (6a)$$

expressing the transfer of neutrons from group i into group j, and

$$\Sigma_{jj} = \sum_{k=1}^{2} I_{jk}^{-1} [\Sigma_a - H]_{kj} , \qquad (6b)$$

expressing the 'effective absorption' in group j.

The distributions $\phi_i(E)$, $\psi_j(E)$ entering Eqs. (1) and (3) are chosen in a general way as the direct and adjoint asymptotic spectra (infinite medium spectra) of the two regions.

The case of two purely scattering moderators is a degenerate one for the outlined formalism, in the sense that being

$$\phi_i(E) = \frac{E}{T_i^2} \exp\left(-\frac{E}{T_i}\right) \equiv M(E,T_i); \quad (i = 1,2)$$

$$\psi_j(E) = \text{const} \qquad\qquad ; \; (j=1,2)$$

the formalism itself is not able to determine the coefficients (6). Therefore we are led to take into account the higher modes of the thermalization operator, and this can be made as follows:

we assume for the nonequilibrium distribution (j) in region (i)

$$\psi_j(E) = 1 + p(E), \qquad (7)$$

being again $\psi_i(E) = 1$. For convenience ψ_j and ψ_i are normalized to unity with respect to $M(E,T_i)$, consequently

$$\int p(E) \; M(E,T_i) \; dE = 0. \qquad (8)$$

The transfer cross sections (6a,b) for region (i) then can be written

$$\Sigma_{ij}^{(i)} = -\frac{\int p(E) \; H^{(i)} \; M(E,T_j) \; dE}{\int p(E) \; M(E,T_j) \; dE} = \Sigma_{jj}^{(i)} \qquad (9a)$$

$$\Sigma_{ji}^{(i)} = 0 = \Sigma_{ii}^{(i)} , \qquad (9b)$$

expressing the fact that the equilibrium group (i) can only gain but not lose neutrons.

We try to determine $p(E)$ by introducing Eq. (7) into the adjoint basic equation written down for the region (i):

$$-\nabla D^{(i)} \nabla \Psi^{(i)}(r,E) - H^{*(i)} \Psi^{(i)}(r,E) = 0 \qquad (10)$$

where H^* is the adjoint thermalization operator defined by the relation

$$(\phi,H^*\psi) = (\psi,H\phi) \equiv \int dE \; \psi(E) \; H\phi(E) \qquad (11)$$

for arbitrary ϕ, ψ. Equation (10) splits up by separation into the following:

$$\nabla^2 [G_i(r) + G_j(r)] = 0 \qquad (10a)$$

$$D^{(i)} \nabla^2 G_j(r) = \mu \; G_j(r) \qquad (10b)$$

$$H^{*(i)} p(E) = -\mu \, p(E). \qquad (10c)$$

According to (10b) the inverse of the eigenvalue μ of the adjoint thermalization operator is a measure for the relaxation of the importance of group (j) decaying in region (i). Since in general the relaxation length connected with the first eigen-

value μ_1 is prevailing, it is justified to introduce in (9a) the first mode $p_1(E)$ of the adjoint thermalization operator. Observing relation (11) we get

$$\Sigma_{ij}^{(i)} = \mu_1^{(i)} \ .$$ (12)

Eq. (12) corresponds to Leslie's result[3]. The results of Selengut[1] and Lindenmeier[2] are obtained by a linear approximation to $p(E)$ satisfying Eq. (8),

$$p(E) \sim (2 T_i - E).$$ (13)

This approximation guarantees both the balance of neutrons and of energy. Moreover the two approaches will agree for higher mass values since (13) is exactly the first mode of the adjoint heavy-gas operator.

Expression (9a) becomes now

$$\Sigma_{ij}^{(i)} = \frac{\int dE \int dE' (E'-E) \ \Sigma_s^{(i)} (E \to E') \ M(E,T_j)}{2(T_i - T_j)}$$ (14)

which can be evaluated analytically for the free gas scattering kernel (mass ratio A, free atom macroscopic scattering cross section Σ_0) yielding

$$\Sigma_{ij}^{(i)} = \frac{2A}{(A+1)^2} \ \Sigma_0^{(i)} \ \left(1 + \frac{T_i}{AT_j}\right)^{1/2} \ .$$ (15)

Pearce[4] has obtained the same formula (15) from a gas-kinetic point of view.

H. Hembd

Joint Nuclear Research Center
Ispra Establishment, Italy

Diffusion Coefficient of Thermal Neutrons for a Maxwellian Gas

The experimental values of the spectra and diffusion parameters of thermal neutrons are usually compared with those calculated on the basis of the free-gas model and on the basis of more realistic models, which account for the effects of the chem-

ical binding (for example, see Ref. 1). In this way, it is possible to have a quantitative insight into the effect of the chemical binding on these values.

Following this line we obtain here an analytical expression for the diffusion coefficient of thermal neutrons for a gas of free protons that are in motion with a Maxwellian distribution of velocities. The diffusion coefficient for nonabsorbing media is defined in general as

$$D(v) = \frac{1}{3N \langle \sigma_s(v) \rangle \ [1 - \mu(v)]} \ ,$$ (1)

where N is the atomic density of the considered medium and $\langle \sigma_s(v) \rangle$ and $\mu(v)$ are the microscopic scattering cross section and the average cosine of the scattering angle in the laboratory system (l.s.), respectively, which are functions of the neutron velocity, v.

As far as the scattering cross section is concerned, we use in Eq. (1) the analytical expression of $\langle \sigma_s(v) \rangle$ given in Refs. 2 and 3.

For the average cosine of the scattering angle in the l.s., $\mu(v)$, we proceed as follows. Under the hypothesis that the scattering cross section $\sigma_0(v_r)$ is independent of the relative velocity v_r of the two colliding particles[3] and the scattering is isotropic in the center of mass system, we may define

$$\mu(v) = \langle \cos\eta(v) \rangle_{\text{eff}} \equiv \frac{\langle \sigma_s(v)\cos\eta(v) \rangle}{\langle \sigma_s(v) \rangle} \ ,$$ (2)

where

$$\langle \sigma_s(v)\cos\eta(v) \rangle$$

$$= \frac{\sigma_0}{v} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-1}^{+1} d\left(\frac{\cos\beta}{2}\right) \int_{-1}^{+1} d\left(\frac{\cos\alpha}{2}\right)$$

$$\times \int_0^{\infty} v_r \cos\eta M_{T_0, M} \ dV.$$ (3)

The notations used in Eq. (3) are those indicated in Ref. 4, pp 101 and 397. Performing the integration appearing on the right-hand side of Eq. (3), one obtains[5] for a free proton gas

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