where F and A are production and destruction operators. The boundary conditions are that ϕ and ϕ^* both vanish on the outer boundary S, and that the initial value of ϕ and the final value of ϕ^* are to be specified.

We now invoke the functional

$$J = \int_{t_a}^{t_b} dt \int_R d\boldsymbol{r}\Big[- (\nabla\phi^*)\cdot D\nabla\phi + \phi^*(\upsilon F - A)\phi - \\ - \frac{1}{2v}\left(\phi^*\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right)\Big] + \\ + \frac{1}{2v}\int_R d\boldsymbol{r}[\phi^*(\boldsymbol{r},t_b)\phi(\boldsymbol{r},t_b) + \phi^*(\boldsymbol{r},t_a)\phi(\boldsymbol{r},t_a)]. \tag{3}$$

The functional used in Ref. 1 does not include the second integral (the boundary terms) on the right side of Eq. (3). The variations with respect to ϕ^* and ϕ yield the Euler equations (1) and (2), the spatial boundary conditions

$$\int_{t_a}^{t_b} dt \int_S dS\, \delta\phi^*\, D\nabla\phi = 0 \tag{4}$$

$$\int_{t_a}^{t_b} dt \int_S dS\, \delta\phi\, D\nabla\phi^* = 0 \tag{5}$$

and the temporal boundary conditions

$$\frac{1}{2v}\int_R d\boldsymbol{r}[\phi(\boldsymbol{r},t_b)\delta\phi^*(\boldsymbol{r},t_b) - \phi(\boldsymbol{r},t_a)\delta\phi^*(\boldsymbol{r},t_a)] + \\ + \frac{1}{2v}\int_R d\boldsymbol{r}[\phi(\boldsymbol{r},t_b)\delta\phi^*(\boldsymbol{r},t_b) + \phi(\boldsymbol{r},t_a)\delta\phi^*(\boldsymbol{r},t_a)] = 0, \tag{6}$$

$$- \frac{1}{2v}\int_R d\boldsymbol{r}[\phi^*(\boldsymbol{r},t_b)\delta\phi(\boldsymbol{r},t_b) - \phi^*(\boldsymbol{r},t_a)\delta\phi(\boldsymbol{r},t_a)] + \\ + \frac{1}{2v}\int_R d\boldsymbol{r}[\phi^*(\boldsymbol{r},t_b)\delta\phi(\boldsymbol{r},t_b) + \phi^*(\boldsymbol{r},t_a)\delta\phi(\boldsymbol{r},t_a)] = 0. \tag{7}$$

In Eqs. (6) and (7), the first integrals arise from integrating the $\frac{\phi}{v}\frac{\partial\delta\phi^*}{\partial t}$ and $\frac{\phi^*}{v}\frac{\partial\delta\phi}{\partial t}$ terms by parts, while the second integrals come from variation of the boundary terms. Since some of the terms cancel, we are left with

$$\frac{1}{v}\int_R d\boldsymbol{r}\,\phi(\boldsymbol{r},t_b)\delta\phi^*(\boldsymbol{r},t_b) = 0 \tag{8}$$

$$\frac{1}{v}\int_R d\boldsymbol{r}\,\phi^*(\boldsymbol{r},t_a)\delta\phi(\boldsymbol{r},t_a) = 0. \tag{9}$$

The functional Eq. (3) is thus stationary with respect to arbitrary variations in the functions ϕ and ϕ^* provided only that the admissible set of functions for ϕ and ϕ^* be restricted to those which satisfy the initial-value conditions imposed on ϕ and the final-value condition imposed on ϕ^* respectively, i.e., that the variations be taken such that

$$\delta\phi(\boldsymbol{r},t_a) = 0 \tag{10}$$

$$\delta\phi^*(\boldsymbol{r},t_b) = 0. \tag{11}$$

Since the adjoint problem is a final-value problem, one specifies final-value conditions in accordance with the interpretation one wishes to assign to the adjoint function[2]. Use of the functional Eq. (3) does not imply knowledge of any function at both end points of the time interval (t_a, t_b).

It is interesting to note that the formal procedure of Ref. 1 is not affected by the inclusion of the boundary terms in the functional Eq. (3). One makes the expansions

$$\phi(\boldsymbol{r},t) = \sum_{k=1}^{K} H_k(\boldsymbol{r})T_k(t) \tag{12}$$

$$\phi^*(\boldsymbol{r},t) = \sum_{k=1}^{K} H_k^*(\boldsymbol{r})T_k^*(t), \tag{13}$$

and one obtains the same set of equations for $T_k(t)$ and $T_k^*(t)$ as one would obtain using the principle of Ref. 1. It is thus seen that the results of Ref. 1 rest on a sounder theoretical base than was originally believed.

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On Cadmium-Ratio Measurements for U^{235} and U^{233} Fission by Fission-Product Gamma Counting

In most determinations of fissile infinite dilution resonance integrals it is necessary to measure a cadmium ratio. One experimental method involves irradiating foils in the spectrum of interest, and subsequently gamma counting the

fission products with a scintillation counter[1,2]; another uses a fission counter to detect fission fragments directly[3]. The latter technique is usually difficult to use because of the physical size of the fission counter. Recently, Bigham[3] suggested that in the gamma-counting technique, differences in thermal and epithermal fission yields would not show up as differences in decay rate if these fission products were short-lived or detected with only low efficiency. To investigate this problem, an experimental study of the U^{235} and U^{233} fission cadmium ratios has been carried out, using both of the methods outlined above.

The experiment was performed using pairs of highly enriched U^{235} (or U^{233}) deposits in a double chamber back-to-back fission counter[4]. Highly enriched U^{235} (or U^{233}) foils were sandwiched between the deposits. This type of counter was used to ensure that the effect due to any flux gradients would be eliminated. Subsequent analysis of the data indicated that this precaution was not necessary. The deposits were 0.1-mg/cm^2 and 0.01-mg/cm^2 thick for U^{235} and U^{233}, respectively, yielding plateaus having a slope of 0.2%/V over a 50-V range. The 0.005-in. (0.013 cm) foils were made from a 2.5 wt% uranium-aluminum alloy enriched to greater than 90% U^{235} (or U^{233}). The self-shielding of these foils was less than 1%.

The irradiation (30 min) was done with the fission counter bare, and repeated with the counter cadmium-covered. The same deposits and foils were used in each set of irradiations to eliminate the need for foil and deposit intercalibrations. Special precautions were taken to ensure that the counter was completely covered to eliminate thermal-neutron streaming paths. This was necessary since the cadmium ratio was approximately 40. The cadmium ratios were determined in two ways for each set of bare and cadmium-covered irradiations. In one, the total fissions from the deposits were recorded during the irradiation; in the other, after the irradiation the fission-product activities of the foils were counted on a 2-in. (5.1 cm) NaI scintillation counter biased to reject pulses below 400 keV. The U^{235} foils were counted between 50 to 100 min after the end of the irradiation, and the U^{233} foils were counted in the interval 30 to 90 min after irradiation. These time intervals are similar to those used in resonance integral determinations[1]. A Cs^{137} standard was used to monitor any drift in the scintillation counter.

Eight sets of cadmium-ratio determinations were obtained for U^{235} and four sets for U^{233}. Corrections were made for counting losses (less than 0.7%), foil self-shielding (less than 0.6%), and for power level drift during the irradiation (less than 1%).

The ratio of the cadmium ratios determined by fission counting to those determined by gamma counting yielded a value of 1.005 ± 0.007 for U^{235} and a value of 1.002 ± 0.005 for U^{233}. These values support the use of the fission-product gamma-counting method in the measurement of U^{235} and U^{233} cadmium ratios, and also establish the validity of this technique as applied to determinations of these fissile infinite dilution resonance integrals.

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Intermediate Resonance Absorption at Low Energies

Is moderator scattering adequately treated in a narrow resonance (*NR*) approximation for low-energy resonances? For cases in which light moderating elements are admixed with the fuel, this may be a particularly important question. Recent work[1,2,3] indicates that the *NR* approximation may not always be applicable to the moderator. The first two references treat this problem for resonance absorption in nonhomogeneous systems; the third concentrates on an examination of the flux shape.

We consider a simple extension of the 'intermediate resonance' (*IR*) formulation[4] of the problem for homogeneous systems. The extension to nonhomogeneous systems may be made by means of equivalence principles[5].

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