limit (in our case, the edge of the continuous spectrum $\mathrm{Re}\lambda = -1$), one can conclude that for the slab, also, the discrete eigenvalues of the IDT problem can exceed this limit. On the other hand, it was stated[13] that these decay constants for the slab cannot exceed the Corngold limit, wherefrom there arises a contradiction.

This contradiction is only virtual, because, *a priori*, the use of IS eigenvalues in computing the IDT ones is not allowed; the spectral analysis of the IDT problem shows that this is not allowed *a posteriori* either. Indeed, for a sphere, it is proved that the integral equation is equivalent with the IDT problem in the whole spectral plane,[11,12] while for the slab, the equivalence is restricted only to the right half-plane, $(\lambda | \mathrm{Re}\lambda > -1)$ (Ref. 14). In the left plane, it is explicitly shown that no eigenvalues (real or complex) can occur and, thus, the IT and, so much the less, the IS equations do not yield eigenvalues with any real significance for the original IDT problem.

4. For anisotropic scattering and slab geometry with vacuum boundary conditions, the question of complex eigenvalues in the IDT problem is not yet solved. This open question must be approached by developing the analysis started by Mika[15] and not by solving some (nonequivalent) IT or IS problems in the left plane $(\lambda | \mathrm{Re}\lambda \leqslant -1)$.

5. As concerns the $\exp(iBr)$ theory, this is formally equivalent with the IDT problem under periodic boundary conditions. Indeed, under periodic boundary conditions,[16] the eigenfunctions have an $\exp(ikx)$ dependence on x; here, k can take a set of discrete values, which lead to different values of λ, according to the eigenvalue equation:

$$1 = \frac{1}{k} \operatorname{arctg} \frac{k}{\lambda + 1} \ ,$$

which is formally identical with the dispersion law,

$$1 = \frac{1}{B} \operatorname{arctg} \frac{B}{\lambda + 1} \ ,$$

obtained from the $\exp(iBr)$ theory. Thus, the role of the buckling parameter, B, which is *fixed* for a given convex body, has no relation with the Fourier parameter k, which, moreover, makes sense only for slab and parallelepipedic geometries. In any case, for both isotropic and anisotropic scattering, the $\exp(iBr)$ theory and the exact solution of the problem under periodic boundary conditions predict a continuous spectrum on the line $(\lambda | \mathrm{Re}\lambda = -1)$, beyond which the dispersion law[2-5] has nothing to do with the IDT problem.

We think that the above remarks clarify some of the aspects existing in the present status of the eigenvalue theory of the Boltzmann equation and can lead to a more adequate approach to the complex eigenvalues problem for anisotropic scattering in slab geometry with vacuum boundary conditions, the only one toward which, in the author's opinion, some effort might be worthy.

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Comments on "On the Eigenvalues of the Stationary and Time-Dependent Boltzmann Equation in Its Integro-Differential and Integral Forms"

The Letter by Protopopescu[1] clarifies many obscure points regarding the existence of eigenvalues to the transport equation. It contains the information I hoped to provoke through an earlier Note.[2]

An important fact in this connection is, of course, that a solution to the integro-differential Boltzmann equation gives the angular-dependent neutron flux density,

$$\psi(x,\mu) = \phi_0(x) + 3P_1(\mu)\phi_1(x) + \dots \ , \qquad (1)$$

whereas the integral equation in its usual form only contains the total neutron flux density $\phi_0(x)$. One of Protopopescu's statements (under his point 3) can therefore be expressed as follows: Even though the integral equation for $\phi_0(x)$ in an infinite slab has eigenvalues beyond the Corngold limit in the case of isotropic scattering, these eigenvalues have no relevance for the complete solution of Eq. (1), which cannot exist in this region.

Some observations in our numerical work[3,4] corroborate the stringent results presented by Protopopescu and shed some further light on the problem. The equivalence between the time-dependent and stationary cases implies that a decay constant beyond the Corngold limit corresponds to a negative total cross section in the stationary case. Since the total cross section enters as a factor into the arguments of the exponential integrals in the matrix elements,[3,5] a negative total cross section means that the exponential integrals diverge. In the case of an infinite slab with vacuum boundary conditions, we have observed the following:

1. Integral equation for ϕ_0, isotropic scattering. The exponential integrals cancel in the matrix elements for the odd harmonic modes. As stated in the earlier Note,[2] this means that eigenvalues can be obtained numerically for ϕ_0 above the Corngold limit.

2. Integral equation for ϕ_0, anisotropic scattering. In this case, it has not been possible to establish an integral equation for the odd harmonic modes of ϕ_0, only an equation coupled to the neutron current density ϕ_1.

3. Integral equation for ϕ_1, isotropic and anisotropic scattering. The exponential integrals seem to persist in the matrix elements. For isotropic scattering and odd harmonic modes, however, the structure of the matrix is such that the exponential integrals do not enter into the calculation of the eigenvalues, only of the eigenvectors. This means that no complete solution of the type of Eq. (1) can be found beyond the Corngold limit. For anisotropic scattering or even harmonic modes, the eigenvalues depend on the exponential integrals.

Regarding the $\exp(iBr)$ theory discussed by Protopopescu, it might be of interest to recall its use in the interpretation of time-dependent experiments with the pulsed neutron source

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method. As is well known (see, e.g., Ref. 6), the introduction of the buckling concept makes it possible to derive diffusion parameters from such experiments. The connection to vacuum boundary conditions is done with the help of extrapolation distances.[7] Since there exist discrete decay constants beyond the Corngold limit for finite bodies, it is natural to try to define appropriate extrapolation distances in this region, also. Although it is not formally correct, this can be done by using the dispersion law. It has been shown to be possible for this purpose to extend the dispersion law curves over the Corngold limit in a simple and straightforward way.[8-10]

Finally, I would like to add that this discussion shows the need for better communication between those who work more strictly mathematically and those who work on the numerical and experimental side. It is commendable that Dr. Protopopescu has made this effort to bridge the gap between these two groups.

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