Letters to the Editor

On the Correct Use of the Bayesian Method for Reactor Core Melt Frequency

There are undoubtedly many ways of interpreting the statistical methodology known as "Bayesian techniques"[1,2] in practice. However, there is general agreement among statisticians who advocate such methods regarding the structural rules for properly conducting a Bayesian analysis. I wish to address the foundational basis of the Bayesian approach used in a recent paper.[3] My contention is that the basic approach used in that paper represented a misapplication of proper Bayesian methods. Let me support my claim.

Most often, the Bayesian method is applied by using opinion (whether expert or otherwise) as a basis for fitting a prior distribution. Objective information (such as observed frequency of core melts for U.S. light water reactors) is then used in the form of a sampling distribution (likelihood) via Bayes' theorem to compute the posterior distribution. What the authors[3] have done is the exact opposite. The objective data are used in computing the prior, while expert opinion (subjective) data are expressed in terms of the likelihood function (or sampling distribution) in computing the posterior. Let me try to demonstrate the consequences of the method they adopted. Suppose one were to fit a gamma prior based on the estimates reported in the *Reactor Safety Study*,[4] the "Rasmussen Report," referred to hereafter as WASH-1400. The two cases considered are

1. a prior distribution fitted using the WASH-1400 5th and 95th percentile estimates
2. a prior distribution fitted based on the WASH-1400 50th and 95th percentile estimates.

Here, zero core melts in 310 reactor years is used as the observed sampling data according to the Poisson distribution. In case 1, the estimates are smaller than those obtained by the authors. This is likely due to the fact that I have not modified the WASH-1400 estimates to reflect the critics' views in fitting the prior distribution. This will be done later. In my case, the prior dominates the resulting posterior estimates, with the small quantity of sampling data having little impact. In the authors' case, the prior distribution is extremely diffuse, and the resulting posterior depends most heavily on the modified WASH-1400 results in their likelihood function.

This observation raises another view. Since their prior is so diffuse, would a classical analysis using their likelihood yield the same basic results as they have obtained? Let us see. As a function of λ, their likelihood [their Eq. (22)] becomes

$$\mathcal{L}(\lambda|\lambda^*) = 1.5 \times 10^{-5} \frac{(6667)^2}{\Gamma(2)} \lambda \exp(-6667\lambda) \ , \ 0 < \lambda < \infty \ . \quad (1)$$

This likelihood can be decomposed as the product of a Poisson distribution, in which $x = 0$ and $T = 6000$, and a gamma "prior" distribution, in which $\alpha_0 = 2$ and $\beta_0 = 6667$. Thus, their likelihood is precisely equivalent to some sort of pseudoexperiment in which zero failures have been observed in 6000 reactor years of operation and, further, in which the failure rate λ follows another gamma "prior" where $\alpha_0 = 2$ and $\beta_0 = 6667$. Let us analyze and compare this gamma "prior" to their posterior estimates. The 5th, 50th, and 95th percentiles, and the mean and mode are easily computed to be 5.3×10^{-5}, 2.5×10^{-4}, 7.1×10^{-4}, 3.0×10^{-4}, and 1.5×10^{-4}, respectively. From these results, it is apparent that the authors' modified likelihood function contributes the substantial portion to their posterior results. In fact, with such a modified likelihood function, there is hardly any benefit in doing a Bayesian analysis at all! One can obtain nearly the same results that the authors get by ignoring the prior completely and simply performing a classical analysis. Their likelihood function is quite diffuse due to the modification of the Poisson "data" ($x = 0$, $T = 6000$) by means of the "prior" ($\alpha_0 = 2$, $\beta_0 = 6667$). That is, the critics' views the authors adopt (which is equivalent to a gamma "prior" for Poisson "data") heavily influence the resulting estimates that they obtain. In fact, their modification "gamma prior" is much stronger than the actual prior that they use. This is intuitively backward. The use of subjective data (critics' views) should yield a "prior" that is somewhat more diffuse than that based on observed data (310 reactor years with zero meltdowns). Most Bayesians will likely agree that such strong subjective arguments properly belong in the prior distribution of a Bayesian analysis and not in the likelihood function.

I now perform the Bayesian analysis correctly using the authors' data. I consider the modified WASH-1400 estimates as prior data and the zero meltdowns in 310 reactor years as Poisson sampling data (via the likelihood) in a Bayesian analysis. Based on the foregoing discussion, a gamma prior distribution is used in which $\alpha' = \alpha_0 + x = 2 + 0 = 2$ and $\beta' = \beta_0 + T = 6667 + 6000 = 12\,667$. Thus, the posterior distribution is gamma with $\alpha'' = 2$ and $\beta'' = \beta' + 310 = 12\,977$. The 5th, 50th, and 95th percentiles are computed to be 2.8×10^{-5}, 1.3×10^{-4}, and 3.7×10^{-4}, respectively, while the mean and mode are 1.5×10^{-4} and 7.9×10^{-5}. It is observed that the results differ from the authors' posterior results (due to their diffuse prior). The 5th percentile estimate is ~180% larger

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[1] B. de FINETTI, *Theory of Probability*, Vols. 1 and 2, John Wiley and Sons, Inc., New York (1974).

[2] R. L. WINKLER, *Introduction to Bayesian Inference and Decision*, Holt, Rinehart and Winston, New York (1972).

[3] G. APOSTOLAKIS and A. MOSLEH, *Nucl. Sci. Eng.*, **70**, 135 (1979).

[4] *Reactor Safety Study*, WASH-1400, U.S. Nuclear Regulatory Commission (1975).

TABLE I

Comparison of the Estimates for the Frequency of Core Melts

Distribution	Comments	5th Percentile	Mode	Median	Mean	95th Percentile
Prior ($\alpha = 0.12, \beta = 120$)	Apostolakis and Mosleh prior	7.4×10^{-14}	---	1.6×10^{5}	1.0×10^{-3}	5.8×10^{-3}
WASH-1400	WASH-1400 results	8.3×10^{-6}	1.5×10^{-5}	5.0×10^{-5}	9.0×10^{-5}	3.0×10^{-4}
Posterior ($\alpha = 1.12, \beta = 6787$)	Apostolakis and Mosleh posterior	1.0×10^{-5}	1.8×10^{-5}	1.2×10^{-4}	1.7×10^{-4}	5.0×10^{-4}
Prior–case 1 ($\alpha = 1.125, \beta = 9177$)	Gamma prior fitted to 5th, 95th percentiles of unmodified WASH-1400 results	8.3×10^{-6}	1.4×10^{-5}	8.9×10^{-5}	1.2×10^{-4}	3.5×10^{-4}
Posterior–case 2 ($\alpha = 0.80, \beta = 10\,310$)	Based on case 2 prior and Poisson data (zero meltdowns in 310 reactor years) in the likelihood	2.1×10^{-6}	---	4.9×10^{-5}	7.8×10^{-5}	2.5×10^{-4}
Prior–case 2 ($\alpha = 0.80, \beta = 10^{4}$)	Gamma prior fitted to 50th, 95th percentiles of unmodified WASH-1400 results	2.2×10^{-6}	---	5.0×10^{-5}	8.0×10^{-5}	2.6×10^{-4}
Posterior–case 1 ($\alpha = 1.125, \beta = 9177$)	Based on case 1 prior and Poisson data (zero meltdowns in 310 reactor years) in the likelihood	8.0×10^{-6}	1.3×10^{-5}	8.6×10^{-5}	1.2×10^{-4}	3.4×10^{-4}
"Prior" ($\alpha_0 = 2, \beta_0 = 6667$)	Modification gamma "prior" based on critics' views of WASH-1400 results	5.3×10^{-5}	1.5×10^{-4}	2.5×10^{-4}	3.0×10^{-4}	7.1×10^{-4}
Posterior ($\alpha' = 2, \beta' = 12\,667$)	Based on modified WASH-1400 results used to fit a gamma prior (using Apostolakis and Mosleh data) and observed Poisson data (zero meltdowns in 310 reactor years) in the likelihood function	2.8×10^{-5}	7.9×10^{-5}	1.3×10^{-4}	1.5×10^{-4}	3.7×10^{-4}

than their posterior estimate, the median estimate is 8% larger than theirs, while the 95th percentile estimate is 26% smaller than their estimate. However, the important difference is that the subjectivity is now in the proper place in the analysis (in the prior), while the sampling data are in their proper place (in the likelihood). This is in conformance with a proper Bayesian approach. Also, a different interpretation of the critics' views would most likely lead to significantly different results. Table I summarizes all of the above estimates, including the authors' estimates, for ease in making rapid comparisons.

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Reply to "On the Correct Use of the Bayesian Method for Reactor Core Melt Frequency"

The basic contention of Martz[1] that the approach of our paper[2] is a misapplication of "proper" Bayesian methods is without foundation. Bayes' theorem is the fundamental tool that allows us to coherently incorporate in our knowledge new evidence, which does not have to be statistical. In fact, we believe that for rare events, such as reactor core meltdowns, the new evidence will almost always come as experts' opinions.

[1]H. F. MARTZ, Jr., *Nucl. Sci. Eng.*, **72**, 368 (1979).
[2]G. APOSTOLAKIS and A. MOSLEH, *Nucl. Sci. Eng.*, **70**, 135 (1979).