B_{wa} = stationary form and viscous drag between wall and phase a
g_x = axial component of acceleration due to gravity
j = mixture volumetric flux = $j_f + j_p = Q/A$
j_f = volumetric flow of phase f
j_p = volumetric flow of phase p
$\dot{m}$ = rate of vapor generation per unit volume
p = thermodynamic pressure
S = slip velocity = $v_x^g - v_x^l = u_f - u_p$
u_a = velocity of phase a
u_t = mixture velocity = $[\theta \rho_f u_f + (1 - \theta)\rho_p u_p]/\rho_t$
V_{fi} = vapor drift velocity = $u_f - j = S(1 - \theta) = S\alpha_l$
v_x^a = velocity of phase a
$\hat{v}$ = intrinsic velocity
v_∞ = terminal velocity
α_a = volume fraction of phase a ($\alpha_l = 1 - \alpha_g$)
ρ = mixture density = $\alpha_g \rho_g + \alpha_l \rho_l = \rho_t$
ρ_a = thermodynamic density of phase a
θ = α_g
$1 - \theta = \alpha_l$

This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory.

Robert W. Lyczkowski

Lawrence Livermore Laboratory
P.O. Box 808
Livermore, California 94550

March 12, 1979

Reply to "Comment on the Drift-Flux Approximation in Transient Two-Phase Flows"

Consider the Lyczkowski[1] Eq. (13) for the relative velocity between phases:

$$\frac{\partial}{\partial t}(u_p - u_f) + \frac{\partial}{\partial x}\left[\frac{1}{2}(u_p^2 - u_f^2)\right] = 0 \ , \tag{13}$$

If we let

$$u_r = u_p - u_f \tag{1}$$

and

$$\bar{u} = \frac{u_p + u_f}{2} \ , \tag{2}$$

then Eq. (13) can be rewritten as

$$\frac{\partial u_r}{\partial t} + \frac{\partial}{\partial x}(u_r \bar{u}) = 0 \tag{3}$$

or

$$\frac{\partial u_r}{\partial t} + \bar{u}\frac{\partial u_r}{\partial x} = -u_r \frac{\partial \bar{u}}{\partial x} \ . \tag{4}$$

Using the well-known "method of Lagrange," we are able to derive a general solution for Eq. (4).

Associated with Eq. (4) is the system of first-order ordinary differential equations:

$$\frac{dx}{dt} = \bar{u} \ , \tag{5}$$

$$\frac{d\bar{u}}{\bar{u}} + \frac{du_r}{u_r} = 0 \ . \tag{6}$$

The solution for Eq. (6) is

$$\bar{u}u_r = c_1 \ . \tag{7}$$

If we assume that the phase velocities differ from each other as

$$u_r = f(x,t) \ , \tag{8}$$

where $f(x,t)$ is a nonzero arbitrary function of x and t, then from Eq. (7), we have

$$\bar{u} = \frac{c_1}{f(x,t)} = g(x,t) \ . \tag{9}$$

The solution for Eq. (5) is then given by

$$h(x,t) = c_2 \ , \tag{10}$$

and the general solution for the system of Eqs. (5) and (6) is therefore

$$\bar{u}u_r = H[h(x,t)] \ . \tag{11}$$

Since Eq. (11) is the general solution of Eq. (4) and our assumption $u_r = f(x,t)$ has not led to a contradiction, the phases can move with a relative velocity that is dependent on both space and time.

J. R. Travis
W. C. Rivard
F. H. Harlow

University of California
Los Alamos Scientific Laboratory
Theoretical Division
Los Alamos, New Mexico 87545

April 9, 1979

[1] ROBERT W. LYCZKOWSKI, *Nucl. Sci. Eng.*, **71**, 77 (1979).

Comments on Neutron-Induced Fission in a Compressed DT-Pu Plasma

Recently, Perkins[1] published two interesting papers about the problem of neutron-induced fission in a compressed plasma composed of deuterium-tritium (DT) seeded with a small amount of ^{239}Pu. The main idea was to produce knock-on deuterium and tritium ions by making use of the collisional energy transfer of the kinetic energy of the fission fragments as they slow down to thermal energies. These suprathermal ions, possessing an average energy of ~5 kT, exhibit an increased fusion probability and hence neutron production. The latter would then couple directly to the fission process. Thus,

---

[1] S. T. PERKINS, *Nucl. Sci. Eng.*, **69**, 137, 147 (1979).

a so-called nuclear fusion chain reaction, whereby the fission fragments serve as catalysts, can be envisioned. The neutron economy in this coupled fission/fusion bootstrap mode ought to be enhanced, which would subsequently increase the energy yield in a microexplosion of a pellet consisting of a mixed DT-Pu fuel.

Although it turned out that the above fusion neutron production can be phenomenologically considered as an inherent part of the fission process (i.e., the fusion neutrons are practically being born instantaneously and at the same spatial location as the fission event), criticality calculations showed that the fusion enhancement is worth only, at most, a factor of 2 reduction in system mass at criticality and, being even more important, the time constant of the neutronic excursion (*e*-folding time) is on the same order as the hydrodynamic disassembly time for such systems ($\sim$1 ns). This latter effect stems from the fact that the prompt neutron lifetime becomes very large due to moderation of the neutrons by the fusion material itself. This is a very disappointing result in view of the applicability of employing homogeneously mixed fusion/fission materials in superprompt critical fuel pellets for the inertial confinement scheme of energy release.

I would like to point out here that we dealt with the same problems during our investigations on the fusion/fission microexplosion technique.[2] The results we have obtained were very similar to the findings of Perkins and were never published. Also, pellets consisting of DT-reflected plutonium cores exhibit prompt neutron system lifetimes that are too large. The corresponding *e*-folding times are too small to reach substantial multiplication. Furthermore, in homogeneously mixed fission/fusion pellets, there is the additional problem of aggravated ignition and enhanced bremsstrahlung losses for optically thin configurations due to the increased Z_{eff}. But there are at least two ways to overcome the above-mentioned problems.

The first way is trivial, namely, to increase the size of the pellet. By doing this, the hydrodynamic disassembly time increases proportionally to the radius while the *e*-folding time does not change appreciably if the same supercritical reactivity is maintained. However, it can easily be seen that one leaves the field of microexplosions very fast.

The second method is more interesting and is related to a radically different pellet design. The idea is to make use of the dynamically stable supersonic propagation of a thermonuclear burn wave along a cylinder of fusion material. This cylinder or thread is ignited at one end, e.g., by an intense laser or particle beam, and the subsequent supersonic heat wave or "thermonuclear flame" is advanced autocatalytically by shock-heating the adjacent cold layers in front of the burning zone by alpha particles, radiation, and thermal conduction of electrons.

In addition to the work by the present author,[3] such a scheme has also been considered by Ahlborn and Stachran[4] and Bobin[5] in the open literature. Winterberg[6] was the first who suggested, on the occasion of the International Conference on Emerging Concepts in Advanced Nuclear Energy Systems in Graz, Austria, March 29-31, 1978, that one could accomplish the isentropic precompression of the cold fuel of a configuration with an increasing cross section by "feeding forward" a part of the soft x rays released in the burning zone. A similar scheme for a cylindrical configuration is also qualitatively described in Ref. 3, but is of no direct interest in this context.

Once ignited, such a cylindrical configuration ought to theoretically possess an infinite pellet gain if the thermonuclear flame will not be quenched after a certain distance and if the configuration is long enough.

My suggestion in connection with the very good work done in the papers by Perkins[1] is to study longish or cylindrical fission/fusion configurations with *alternate* (and not homogeneously mixed) fuel zones of fissile and fusionable materials. The question is whether or not suprathermal knock-on fusion ions produced in a pure fusion zone, immediately behind a very thin foil of fissionable material, can enhance the velocity of the thermonuclear burn of deflagration wave within the configuration. It may be that there will be some positive effect, particularly if one takes into account not only the alpha-particle heating but also the neutron heating effect, the latter being present if the length of the configuration is larger than the mean-free-path of the neutrons. The burning zone produces fast neutrons, and a part of them will be slowed down by the light elements of the fusion fuel ahead of the burning front, thereby heating it. The thermalized neutrons may now be used to produce fission events in thin fissionable foils. If the foil is thin enough, half of the fission fragments will escape in the fusion zone ahead of the foil. Both suprathermal knock-on and fully thermalized ions are generated, which subsequently could start a new burn front. Thus, "energetically worthless" thermalized neutrons could be converted into localized heat in the fusion fuel, giving rise to an enhanced fusion neutron production. Furthermore, the production on intense soft x rays in the high-*Z* fissionable zone could also have a certain influence. Using metaphorical language, one could call such a phenomenon a "neutronic relay" rather than the mixed fusion or fission chain reaction described by Perkins. The overall effect of this relay could be that the velocity of propagation of a thermonuclear burn front may be additionally accelerated; hence, the overall energy yield is substantially increased.

W. Seifritz

Swiss Federal Institute for Reactor Research
CH-5303 Würenlingen
Switzerland

March 28, 1979

---

[2] W. SEIFRITZ and J. LIGOU, *Trans. Am. Nucl. Soc.*, **18**, 18 (1974).

[3] W. SEIFRITZ, *Atomkernenergie*, **32**, *1*, 56 (1978).

[4] B. AHLBORN and J. D. STACHRAN, *Can. J. Phys.*, **51**, 1416 (1973).

[5] J. L. BOBIN, "Nuclear Fusion Reactions in Fronts Propagating in Solid DT," Presented at the Third Workshop on Laser Interactions and Related Plasma Phenomena, held at Rensselaer Polytechnic Institute, Troy, New York, August 13-17, 1973.

[6] F. WINTERBERG, *Atomkernenergie*, **32**, *2*, 85 (1978).