Letters to the Editor

Comment on the Drift-Flux Approximation in Transient Two-Phase Flows

INTRODUCTION

The drift-flux approximation to two-fluid, two-phase flows is examined in light of recent attempts to extend it to multidimensions. It is found that inconsistent approximations may result in anomalous results.

A recent paper by Travis et al.[1] presented a derivation of the drift-flux approximation to the two-fluid, two-phase flow equations useful for multidimensional flows. This is perhaps the first literature publication to do so, although an earlier report[2] presented a relative velocity expression for use in their two-dimensional computer program. The purpose of this Letter is to briefly examine the drift-flux theory to place the approximation in perspective. The approximate drift-flux momentum field equations developed by Travis et al.[1] are analyzed and found to predict an anomalous result.

ANALYSIS

The bases of the drift-flux field equations were examined by Lyczkowski, who showed explicitly what simplifying assumptions are made in obtaining the generally accepted[3-5] transient, one-dimensional drift-flux model of two-phase flow from the two-fluid or separated flow model.[6] Sometimes referred to as the "diffusion model,"[7] the four-equation one-dimensional drift-flux model field equations presented by Wulff et al.,[5] consisting of conservation of mixture mass, vapor mass, and mixture momentum and energy, were shown to be algebraically equivalent to the two-fluid model. The assumption of limited thermal equilibrium (vapor or liquid saturated) allows one energy equation to be dropped. Except for slightly different notation and structure, these differential equations agree with those written by Hirt and Romero[2] in one dimension.

Although the form and notation used differ from one author to another, they all have in common a mixture momentum equation similar to Eq. (21) of Ref. 1. This mixture momentum equation is normally written in terms of the relative velocity or in terms of the drift velocity (using the notation of Ref. 1 in one dimension) as

$$\frac{\partial}{\partial t}(\rho_t u_t) + \frac{\partial}{\partial x}(\rho_t u_t u_t) + \frac{\partial}{\partial x}\left[\left(\frac{\rho_p - \rho_t}{\rho_t - \rho_f}\right)\frac{\rho_p \rho_f}{\rho_t} V_{fj}^2\right] - g_x \rho_t + \frac{\partial p}{\partial x} = 0 \ , \tag{1}$$

where V_{fj} is the drift velocity, defined as[3,4]

$$V_{fj} = u_f - j \ . \tag{2}$$

Here, j_f is the volumetric flux (or superficial velocity) of the fluid phase, defined as

$$j_f = \theta u_f \ , \tag{3}$$

and j is the volumetric average velocity, defined as

$$j = j_f + j_p = \theta u_f + (1 - \theta) u_p = \frac{Q}{A} \ . \tag{4}$$

The term Q is the volumetric flow, and A is the area. The relative velocity is related to the drift velocity by

$$V_{fj} = \frac{u_f - u_p}{\theta} = S(1 - \theta) \ , \tag{5}$$

where S is the relative velocity.

The drift velocity is then measured and correlated by purely algebraic expressions[3,8] and replaces the second constituent momentum equation. Zuber and Staub,[3,8] for example, give the general expression for the drift flux for vertical dispersed flow as

$$V_{fj} = v_\infty (1 - \theta)^m \ . \tag{6}$$

Starting with a representative set of two-fluid equations, Lyczkowski[6] showed that a general form of the transient velocity difference equation is given by

---

[1]J. R. TRAVIS, F. H. HARLOW, and A. A. AMSDEN, *Nucl. Sci. Eng.*, **61**, 1 (1976).

[2]C. W. HIRT and N. C. ROMERO, "Application of a Drift-Flux Model to Flashing in Straight Pipes," LA-6005-MS, Los Alamos Scientific Laboratory (1975).

[3]NOVAK ZUBER and F. W. STAUB, *Int. J. Heat Mass Transfer*, **9**, 871 (1966).

[4]NOVAK ZUBER, "Flow Excursions and Oscillations in Boiling Two-Phase Flow Systems with Heat Addition," *Proc. Symp. Two-Phase Flow Dynamics*, **1**, 1071, Euratom, Brussels (June 1965).

[5]WOLFGANG WULFF et al., "Development of a Computer Code for Thermal-Hydraulics of Reactors (THOR)," Second Quarterly Progress Report, BNL-50455, Brookhaven National Laboratory (1975).

[6]R. W. LYCZKOWSKI, "Theoretical Bases of the Drift-Flux Field Equations and Vapor Drift Velocity," *Proc. Sixth Int. Heat Transfer Conf.*, Toronto, August 7-11, 1978, CONF-780807 800, Vol. 1, p. 339, Hemisphere Publishing Corporation, Washington, D.C. (1978).

[7]M. ISHII, *Thermo-Fluid Dynamic Theory of Two-Phase Flow*, Eyrolles, Paris (1975).

[8]NOVAK ZUBER and F. W. STAUB, *Nucl. Sci. Eng.*, **30**, 268 (1967).

$$S = \frac{\alpha_g \alpha_l}{\bar{A}_{gl} B_{gl}} \left[\rho_l \frac{\partial}{\partial t} v_x^l + \rho_l v_x^l \frac{\partial}{\partial x} v_x^l - \rho_g \frac{\partial}{\partial t} v_x^g - \rho_g v_x^g \frac{\partial}{\partial x} v_x^g \right]$$
$$+ \alpha_g \frac{\bar{A}_{wl} B_{wl}}{\bar{A}_{gl} B_{gl}} v_x^l - \alpha_l \frac{\bar{A}_{wg} B_{wg}}{\bar{A}_{gl} B_{gl}} v_x^g + \frac{a_g \dot{m}}{\bar{A}_{gl} B_{gl}} (\hat{v} - v_x^l)$$
$$+ \frac{\alpha_l \dot{m}}{\bar{A}_{gl} B_{gl}} (\hat{v} - v_x^g) + \frac{\alpha_g \alpha_l}{\bar{A}_{gl} B_{gl}} (\rho_g - \rho_l) g_x \ . \quad (7)$$

The terms in brackets in Eq. (7) above represent acceleration. The remaining terms account for wall friction of the phases, momentum transfer caused by mass transfer, and gravity. An association with the notation of Travis et al.[1] can be made with the following equivalences:

$$\begin{aligned} \theta &\to \alpha_g \ , \\ 1 - \theta &\to \alpha_l \ , \\ \rho_p &\to \rho_l \ , \\ \rho_f &\to \rho_g \ , \\ \rho &\to \rho_t \ , \\ v_x^l &\to u_p \ , \\ v_x^g &\to u_f \ , \\ \bar{A}_{gl} B_{gl} &\to K \ . \end{aligned} \quad (8)$$

Equation (7), with all terms on the right side dropped except the last one, is the theoretical basis of all of the empirical vapor drift velocity expressions, including Eq. (6) above. Acceptable vapor drift velocity correlations are presently only available for steady-state gravity-dominated flow in vertical ducts.[9]

An alternative but completely equivalent expression for the transient velocity difference can be obtained as[10,11]

$$\frac{\partial S}{\partial t} + v_x^g \frac{\partial v_x^g}{\partial x} - v_x^l \frac{\partial v_x^l}{\partial x} + \left(\frac{\rho_l - \rho_g}{\rho_l \rho_g}\right) \frac{\partial p}{\partial x} + \frac{\rho \bar{A}_{lg} B_{lg} S}{\alpha_l \alpha_g \rho_l \rho_g} + \frac{\bar{A}_w^g B_w^g v_x^g}{\rho_g \alpha_g}$$
$$- \frac{\bar{A}_w^l B_w^l v_x^l}{\rho_l \alpha_l} = \dot{m} \left[\left(\frac{\hat{v} - v_x^g}{\alpha_g \rho_g}\right) - \left(\frac{\hat{v} - v_x}{\alpha_l \rho_l}\right) \right] \ . \quad (9)$$

In this form, the explicit appearance of the gravity term vanishes, but the pressure gradient appears. If all terms in Eq. (9) are dropped except the fourth and fifth ones on the left side, then the one-dimensional analog of Eq. (A.10) in Ref. 2 or Eq. (23) of Ref. 1 is obtained. This equation should properly replace one of the two momentum equations as

$$S = \frac{\alpha_l \alpha_g (\rho_g - \rho_l)}{\bar{A}_{lg} B_{lg} \rho} \frac{\partial p}{\partial x} \ . \quad (10)$$

Equation (10) resembles Darcy's law for flow through porous media.[12] In one dimension, flow regime maps are used to select the $\bar{A}_{lg} B_{lg}$ product[13] (the K drag function of Refs. 1 and 2). For multidimensional flow, both Refs. 1 and 2 assumed dispersed-type flow for this drag function.

Travis et al.[1] substitute their analog of Eq. (10) above back into the component momentum equations to arrive at (in one dimension in their notation)

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = g_x - \frac{1}{\rho_t} \frac{\partial p}{\partial x} \quad (11)$$

and

$$\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} = g_x - \frac{1}{\rho_t} \frac{\partial p}{\partial x} \ . \quad (12)$$

This use of the drift-flux approximation is thus seen to be quite unlike the usual treatment outlined above.

Equations (11) and (12) admit a solution that predicts that the phases will never move at a relative velocity different from the initial value. The following example illustrates this observation.

First subtract Eqs. (11) and (12) to obtain

$$\frac{\partial}{\partial t} (u_p - u_f) + \frac{\partial}{\partial x} \left[\frac{1}{2} (u_p^2 - u_f^2) \right] = 0 \ . \quad (13)$$

Then, assume that the phase velocities differ from each other as

$$u_p(x,t) - u_f(x,t) = k \ , \quad (14)$$

where k is an arbitrary constant to be determined.

Assumption (14) together with Eq. (13) implies that u_f and u_p are at most functions of time of the form

$$u_f(t) = f_1(t) \ , \quad (15)$$
$$u_p(t) = k + f_1(t) \ . \quad (16)$$

Apply the initial conditions consistent with Eq. (14) as

$$u_f(0) = f_1(0) \ , \quad (17)$$
$$u_p(0) = k + f_1(0) \ . \quad (18)$$

Then, k is obtained as

$$k = u_p(0) - u_f(0) \ , \quad (19)$$

which implies that

$$u_f(x,t) - u_p(x,t) = u_p(0) - u_f(0) \ . \quad (20)$$

Since k was picked to be arbitrary, Eq. (20) holds for all constant values. Thus, the original assumption is not contradicted, and the phases never move at a relative velocity other than that prescribed at zero time.

NOMENCLATURE

A = area

$\bar{A}_{gl}$ = surface area between vapor and liquid phase per unit volume

$\bar{A}_{wa}$ = surface area of phase a on contact with the wall per unit volume

B_{gl} = friction coefficient between vapor and liquid phases

[9] W. WULFF et al., "Development of a Computer Code for Thermal-Hydraulics of Reactors (THOR)," First Quarterly Progress Report, BNL-19978, Brookhaven National Laboratory (1975).

[10] R. W. LYCZKOWSKI, D. C. MECHAM, A. A. IRANI, N. FUJITA, G. R. SAWTELLE, and K. V. MOORE, "The Development of RELAP/SLIP for the Semiscale Blowdown Heat Transfer Test S-02-6 (NRC Standard Problem 6), Interim Report NP-343, Electric Power Research Institute (1976).

[11] R. W. LYCZKOWSKI, D. C. MECHAM, and C. W. SOLBRIG, "RELAP/SLIP–A General Purpose One-Dimensional Two-Fluid Thermo-hydraulic Computer Program," *Proc. Sixth Int. Heat Transfer Conf.*, Toronto, August 7-11, 1978, CONF-780807 800, Vol. 5, p. 77, Hemisphere Publishing Corporation, Washington, D.C. (1978).

[12] A. E. SCHEIDEGGER, *The Physics of Flow Through Porous Media*, 3rd. ed., University of Toronto Press (1974).

[13] C. W. SOLBRIG, J. H. McFADDEN, R. W. LYCZKOWSKI, and E. D. HUGHES, "Heat Transfer and Friction Correlations Required to Describe Steam-Water Behavior in Nuclear Safety Studies," *Heat Transfer: Research and Application*, AIChE Symp. Series No. 174, **74**, 100, American Institute of Chemical Engineers, New York (1978).

B_{wa} = stationary form and viscous drag between wall and phase a

g_x = axial component of acceleration due to gravity

j = mixture volumetric flux = $j_f + j_p = Q/A$

j_f = volumetric flow of phase f

j_p = volumetric flow of phase p

$\dot{m}$ = rate of vapor generation per unit volume

p = thermodynamic pressure

S = slip velocity = $v_x^g - v_x^l = u_f - u_p$

u_a = velocity of phase a

u_t = mixture velocity = $[\theta \rho_f u_f + (1 - \theta)\rho_p u_p]/\rho_t$

V_{fi} = vapor drift velocity = $u_f - j = S(1 - \theta) = S\alpha_l$

v_x^a = velocity of phase a

$\hat{v}$ = intrinsic velocity

v_∞ = terminal velocity

α_a = volume fraction of phase a ($\alpha_l = 1 - \alpha_g$)

ρ = mixture density = $\alpha_g \rho_g + \alpha_l \rho_l = \rho_t$

ρ_a = thermodynamic density of phase a

θ = α_g

$1 - \theta = \alpha_l$

This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory.

Robert W. Lyczkowski

Lawrence Livermore Laboratory
P.O. Box 808
Livermore, California 94550

March 12, 1979

Reply to "Comment on the Drift-Flux Approximation in Transient Two-Phase Flows"

Consider the Lyczkowski[1] Eq. (13) for the relative velocity between phases:

$$\frac{\partial}{\partial t}(u_p - u_f) + \frac{\partial}{\partial x}\left[\frac{1}{2}(u_p^2 - u_f^2)\right] = 0 \ , \tag{13}$$

If we let

$$u_r = u_p - u_f \tag{1}$$

and

$$\bar{u} = \frac{u_p + u_f}{2} \ , \tag{2}$$

then Eq. (13) can be rewritten as

$$\frac{\partial u_r}{\partial t} + \frac{\partial}{\partial x}(u_r \bar{u}) = 0 \tag{3}$$

or

$$\frac{\partial u_r}{\partial t} + \bar{u}\frac{\partial u_r}{\partial x} = -u_r \frac{\partial \bar{u}}{\partial x} \ . \tag{4}$$

Using the well-known "method of Lagrange," we are able to derive a general solution for Eq. (4).

Associated with Eq. (4) is the system of first-order ordinary differential equations:

$$\frac{dx}{dt} = \bar{u} \ , \tag{5}$$

$$\frac{d\bar{u}}{\bar{u}} + \frac{du_r}{u_r} = 0 \ . \tag{6}$$

The solution for Eq. (6) is

$$\bar{u}u_r = c_1 \ . \tag{7}$$

If we assume that the phase velocities differ from each other as

$$u_r = f(x,t) \ , \tag{8}$$

where $f(x,t)$ is a nonzero arbitrary function of x and t, then from Eq. (7), we have

$$\bar{u} = \frac{c_1}{f(x,t)} = g(x,t) \ . \tag{9}$$

The solution for Eq. (5) is then given by

$$h(x,t) = c_2 \ , \tag{10}$$

and the general solution for the system of Eqs. (5) and (6) is therefore

$$\bar{u}u_r = H[h(x,t)] \ . \tag{11}$$

Since Eq. (11) is the general solution of Eq. (4) and our assumption $u_r = f(x,t)$ has not led to a contradiction, the phases can move with a relative velocity that is dependent on both space and time.

J. R. Travis
W. C. Rivard
F. H. Harlow

University of California
Los Alamos Scientific Laboratory
Theoretical Division
Los Alamos, New Mexico 87545

April 9, 1979

[1] ROBERT W. LYCZKOWSKI, *Nucl. Sci. Eng.*, **71**, 77 (1979).

Comments on Neutron-Induced Fission in a Compressed DT-Pu Plasma

Recently, Perkins[1] published two interesting papers about the problem of neutron-induced fission in a compressed plasma composed of deuterium-tritium (DT) seeded with a small amount of ^{239}Pu. The main idea was to produce knock-on deuterium and tritium ions by making use of the collisional energy transfer of the kinetic energy of the fission fragments as they slow down to thermal energies. These suprathermal ions, possessing an average energy of ~5 kT, exhibit an increased fusion probability and hence neutron production. The latter would then couple directly to the fission process. Thus,

[1] S. T. PERKINS, *Nucl. Sci. Eng.*, **69**, 137, 147 (1979).