The characteristics λ_1 and λ_2 represent the sound-wave characteristics along which small-amplitude pressure disturbances propagate. The third characteristic, λ_3, represents a slow-wave characteristic along which the velocity-difference or phase-slip disturbance propagates.

If $d\alpha/dp < 0$, $\lambda_{1,2}$ will be real. Consider the case of thermal equilibrium, then the vapor void fraction α is a function of the pressure and mixture specific enthalpy h. For a flow with constant enthalpy ($dh = 0$), α is a function of pressure only. In this case, $d\alpha/dp$ is given by

$$\frac{d\alpha}{dp} = - \frac{\frac{\rho}{\rho_g} F + \frac{\chi\rho}{\rho_g^2 a_g^2} - \frac{x}{\rho_g}\left(\frac{\alpha_g}{a_g^2} + \frac{\alpha_f}{a_f^2}\right)}{1 + \frac{x}{\rho_g}(\rho_f - \rho_g)} , \tag{11}$$

where

$$F = \frac{1}{h_{gs} - h_{fs}} \frac{dh_{fs}}{dp} + \frac{h - h_{fs}}{(h_{gs} - h_{fs})^2}\left(\frac{dh_{gs}}{dp} - \frac{dh_{fs}}{dp}\right),$$

$$h = xh_g + (1 - x)h_f ,$$

$$\rho = \alpha_g\rho_g + \alpha_f\rho_f ,$$

$$x = \frac{h - h_{fs}}{h_{gs} - h_{fs}} ,$$

and

$$\alpha = \frac{\chi\rho}{\rho_g} , \tag{12}$$

and the subscript s denotes saturation line. Here, h is the specific enthalpy of phase k and x is the static quality.

Using the fact that $a_g < a_f$, $\rho \geqslant \rho_g$, and $F > 0$, it is clear that $d\alpha/dp < 0$. Therefore, the two-phase flow equations are always hyperbolic for the case of thermal equilibrium. Since the equation set for the homogeneous nonequilibrium model is always hyperbolic, we would expect that $d\alpha/dp < 0$ is true also for the thermal nonequilibrium case. Hence, if the phase flow area is taken to be a function of phase pressure, the two-phase flow equation set is always hyperbolic for the case of equal phase pressures.

However, if the phase flow area is not explicitly dependent on pressure and/or other variables (i.e., there is no constraint on α), the characteristics for Eqs. (1) and (7) are

$$\lambda_{1,2} = u \pm a \tag{13}$$

and

$$\lambda_{3,4} = v \pm w , \tag{14}$$

where

$$u = \frac{\alpha_g\rho_f u_g + \alpha_f\rho_g u_f}{\alpha_f\rho_g + \alpha_g\rho_f} ,$$

$$a^2 = \frac{\alpha_f\rho_g + \alpha_g\rho_f}{\frac{\alpha_f\rho_g}{a_f^2} + \frac{\alpha_g\rho_f}{a_g^2}} ,$$

$$v = \frac{\alpha_f\rho_g u_g + \alpha_g\rho_f u_f}{\alpha_g\rho_f + \alpha_f\rho_g} ,$$

and

$$w = i\,\frac{(u_g - u_f)(\alpha_g\alpha_f\rho_g\rho_f)^{1/2}}{\alpha_g\rho_f + \alpha_f\rho_g} .$$

It is clear that the equation set is not hyperbolic unless $u_g = u_f$. The equation set can be hyperbolic only when other static and/or dynamic forces are included in the momentum equations. For example, the gravity force, interfacial pressure term, surface tension, viscous term, and the virtual mass force, which constrain the phase pressure, void fraction, or phase velocities, will render the equation set hyperbolic. As a result of the constraints on phase pressure, void fraction, and phase velocities, which occur physically, additional couplings between two phases occur.

It can be concluded that the constraint on the phase pressure results in a change in the sound-wave characteristics (i.e., two pairs of sound waves become a pair of sound wave) and constraints on volume fraction and phase velocities modify the slow-wave characteristics of the two-fluid model. Complex characteristics of the equation set result when the physical constraints on the volume fraction and phase velocities are ignored.

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Reply to "Comment on the Nonhyperbolicity of Two-Phase Flow Equations"

The analysis[1] may contain elements of a contradictory nature for the following reasons.

1. Liu used a different equation-of-state, $\rho_k = \rho_k(p)$, to arrive at his Eqs. (6) and (17). This reduced the order of the system from four to three and he obtained different characteristics.

2. The quantity $d\alpha/dP$ is not always negative. The most obvious situation is the decompression of an initially saturated vapor that causes condensation to occur (Wilson cloud chamber effect). The value $a_f^{-2} = d\rho_f/dP$ is negative along the saturation line.

3. Lyczkowski et al.[2] found that Eqs. (1) and (7) produce a characteristic polynomial which does not factor. If $a_f^{-2} = a_g^{-2} = 0$, then Eq. (14) results, but λ_{12} are infinite.

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[1]W. S. LIU, *Nucl. Sci. Eng.*, **78**, 305 (1981).

[2]ROBERT W. LYCZKOWSKI, DIMITRI GIDASPOW, CHARLES W. SOLBRIG, and E. D. HUGHES, *Nucl. Sci. Eng.*, **66**, 378 (1978).